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SEPTEMBER, 1928

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THE USE OF MOVING AVERAGES IN THE MEASUREMENT OF SEASONAL VARIATIONS

By Aryness Joy and Woodlief Thomas, *Division of Research and Statistics, Federal Reserve Board*

Seasonal variations—long a commonplace to practical business men—offer to statisticians a perplexing problem of measurement. Business men commonly make allowance for the usual seasonal changes by some rule-of-thumb process for which the statistician has sought to substitute a more nearly exact set of measurements which would be of universal service in analyzing current trends and predicting the future. Changes in economic phenomena, however, being the result of an infinite number of forces, do not follow set paths with mathematical rigidity. Therefore, the statistician's task has been the difficult one of discovering a method which allows for constantly changing conditions. As a result, statistical devices to discover and measure the magnitude of those movements which recur within each year have increased in number and in the ingenuity of their refinements. These various devices furnish at best no more than mere approximations which must be revised occasionally to take account of gradual changes that affect the character of seasonal variations.

It is the purpose of this article to present the results of the Federal Reserve Board's experiments with methods of measuring seasonal variations in over one hundred series comprising its revised indexes of production and trade, and to describe the ratio-to-moving-average method which was adopted for use.[1] This method commends itself because of simplicity of conception, rendering it more intelligible to users of index numbers, and because of ease of computation, reasonable

[1] Descriptions of these index numbers are to be found in the following issues of the *Federal Reserve Bulletin:* Industrial production, February, 1927, pp. 100–103, and March, 1927, pp. 170–177; building contracts, freight-car loadings, and revised index of automobile production, August, 1927, pp. 562–563; wholesale distribution, December, 1927, pp. 817–828; department-store sales and stocks, February, 1928, pp. 114–124; revised indexes of chain-store and mail-order house sales, April, 1928, pp. 236–242.

accuracy, and flexibility to changing conditions. Its adaptability to the measurement of both constant and changing types of seasonal variations makes it doubly useful for the analysis of economic series.

The conception of this method has been attributed to Dr. Fred R. Macaulay, of the National Bureau of Economic Research. In adapting it the writers have also drawn upon methods devised by Mr. Rollo H. Britten and Mr. Edgar Sydenstricker, of the United States Public Health Service, and Dr. Willford I. King, of the National Bureau of Economic Research.[1] It relies upon a twelve-month moving average as a rough measure of long-time growth and the larger short-time, non-seasonal fluctuations. These influences, being measured, are then removed from the data in order to define more clearly the seasonal factors. The details of its application to series in which seasonal fluctuations appear to be fairly constant over a period of years will be illustrated by the computation of seasonal factors for the production of cement.

CONSTANT SEASONAL FACTORS

The average daily output of cement from 1913 to June, 1927, shown in Chart I, furnished the data for these computations. To obtain this series, which places the months on an equality with respect to the *rate* of production, the total monthly output of cement was divided by the number of working days in the month. This removed the fluctuations attributable to the arrangement of the Gregorian calendar. In the case of cement, which is produced by a continuous manufacturing process, this adjustment was made on the basis of total number of days in each month; but in those industries which close on Sundays, the varying number of Sundays in the month was also taken into account. Allowances were likewise made for the observance of holidays. The effect of differences in number of working days, which are not, strictly speaking, "seasonal" in nature, is direct and measurable and is removed by this adjustment.[2]

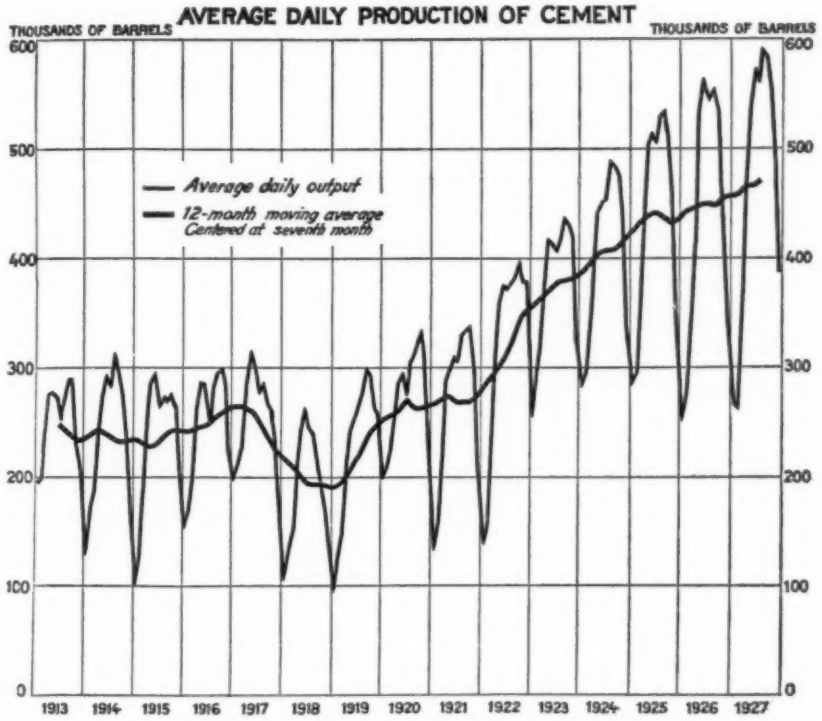
The steps in the process of deriving measures of seasonal variations were as follows:

(1) A twelve-month moving average of these average daily figures

[1] See "An Improved Method for Measuring Seasonal Factors," by Willford I. King, this JOURNAL, September, 1924, pp. 301–313.

[2] In many series there are regular seasonal fluctuations within the calendar week, and variations in the number of a particular day of the week included in the month cause fluctuations in monthly totals which are difficult to measure because of the lack of daily statistics. In the case of retail trade, for example, sales are undoubtedly larger on Saturdays than on other days of the week in rural communities, especially during the crop-moving season, while in cities during the summer, sales are probably smallest on Saturdays, as evidenced by practice of many large department stores to close on that day. These influences are definitely of a seasonal nature, but are incapable of being measured because of inadequacies of data.

CHART I



was computed and placed opposite the seventh month of its set of twelve. The method of computation is shown in Table I. The

TABLE I

COMPUTATION OF SEASONAL FACTORS FOR CEMENT*

(In thousands of barrels)

Year and month (1926)	(A) Average daily output	(B) Twelve-month moving total of (A) centered at seventh month	(C) Twelve-month moving average centered at seventh month [(B)÷12]	(D) Ratio of average daily production to moving average [100(A)÷(C)]
January	254.4	5,254.8	437.9	58.1
February	276.1	5,301.8	441.8	62.5
March	334.0	5,318.5	443.2	75.4
April	413.4	5,339.6	445.0	92.9
May	531.4	5,359.1	446.6	119.0
June	560.9	5,377.6	448.1	125.2
July	551.5	5,375.5	448.0	123.1
August	546.3	5,387.5	449.0	121.7
September	552.4	5,377.8	447.9	123.3
October	535.4	5,306.9	442.2	121.1
November	473.1	5,262.9	438.6	107.9
December	346.6	5,199.8	433.3	80.0

* Figures for the months from July to December, 1925, and from January to June, 1927, used in computing the first six and the last five moving totals and averages, are as follows: 1925—July, 504.5, August, 529.6, September, 531.3, October, 515.9, November, 454.6, December, 348.7; 1927—January, 266.4, February, 263.1, March, 369.4, April, 468.3, May, 537.9, June, 572.2.

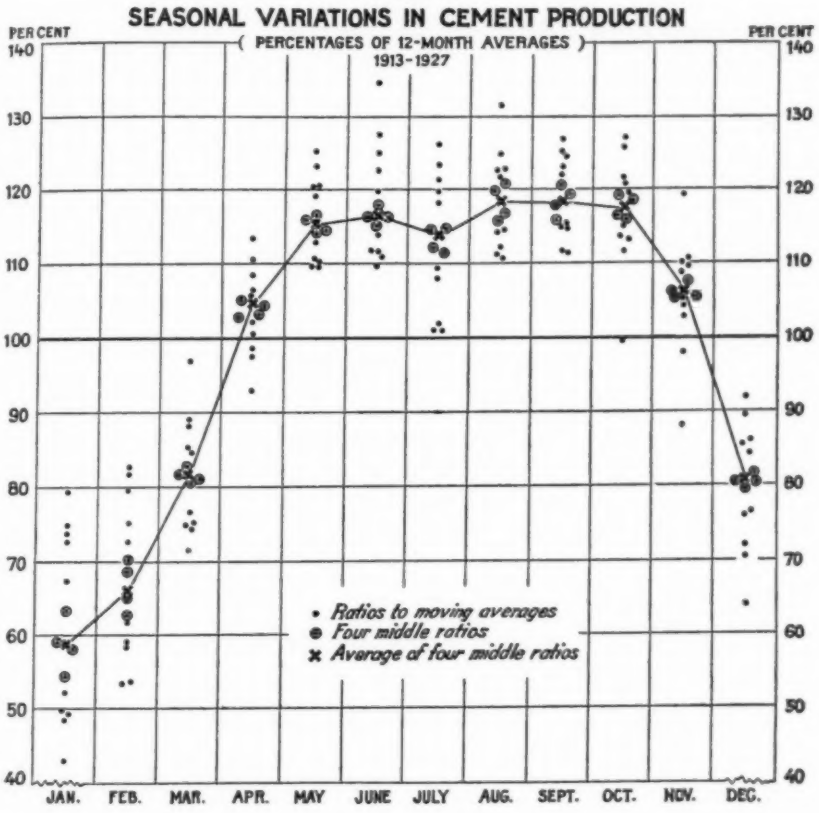
average daily output of cement for the twelve months of 1926 (Column A), for example, was totaled, averaged, and placed opposite July; the next average, for the twelve months from February, 1926, through January, 1927, was placed opposite August of 1926, and in this manner the average, commencing with the first twelve months of data, moved on throughout the series. (See Columns B and C, Table I.) The resulting curve, shown on Chart I, is a relatively smooth one which reflects the more important non-seasonal fluctuations such as the long-time growth of the industry, the drop in production in 1917 and 1918, and the depression of 1921.

(2) The influence of the non-seasonal fluctuations measured in this manner was then removed by dividing the figures of average daily output for each month by the moving average placed opposite it (and multiplying by 100 to remove the decimals). (See Table I, Column D.) The resulting relatives indicate, then, the relation of the actual production of each month to a base which reflects the major changes in the industry, but which does not reflect seasonal variations since it is an average of all months in the year. Comparison of the figure for a given month with a moving average for twelve months, of which the particular month is practically the center, is preferable to comparison with a calendar-year average, because in the former the monthly increment of growth has been removed from the resulting ratios, whereas in the latter all of the influence of long-time growth occurring within the entire year would be reflected in the difference between the ratios for December and the following January. In a series, such as cement, where growth has been rapid, the ratios for December would always be larger than those for January. In a similar gradual manner the moving average reflects the influences of cyclical and other movements which extend beyond the arbitrary limits of a calendar year.

(3) The ratios were next arranged in monthly groups in order of size without regard to chronology. The clustering of these ratios about a central value in each month (see Chart II) indicates that the output of cement in that month ordinarily approximated a certain proportion of the annual total throughout most of the period studied.

(4) In order to derive an average figure representative of each monthly group, averages of the four middle items of these arrays were computed. In other economic series the number of items used in the average varied in accordance with the length of time for which data were available and also according to whether the number of years was even or odd. For example, in the case of pig-iron production, for which there were data for the twenty-six years from 1902 to 1927, the average of six middle items was used, whereas for those production series avail-

CHART II



able for only from six to ten years, two items were used if the number of years was even and three if the number was odd. The crossmark (x) shown for each month on Chart II represents the typical figure of each array, derived by averaging the four middle items, which are indicated by dots surrounded by circles. The averages of the four middle items are also given in the first column of Table II. The selection of only the middle items, instead of an average of all the ratios, eliminates the extreme items, and, on the other hand, is preferable to a median, for in a small array the accidental distribution of the values frequently gives a median which is not typical.

(5) These twelve average ratios were next adjusted to total 1,200 by prorating the difference between their sum and 1,200 in proportion to the size of each ratio. These figures were then rounded off to whole numbers. (Table II, Column 2.) The resulting relatives are preliminary measures of seasonal variations and represent the propor-

TABLE II

MONTHLY MEASURES OF SEASONAL VARIATIONS IN CEMENT PRODUCTION

Month	(1) Average of four middle ratios *	(2) Adjusted to equal 1,200	(3) Seasonal factors
January	58.6	59	60
February	66.7	67	67
March	81.4	82	81
April	104.2	104	104
May	115.2	115	115
June	116.7	117	117
July	113.2	113	114
August	118.5	119	118
September	118.5	119	119
October	117.7	118	118
November	106.3	106	106
December	80.4	81	81
	1,197.0	1,200	1,200

* Ratios of actual average daily production in each month to corresponding monthly figure of a twelve-month moving average, centered at the seventh month. See Table I, Column (D).

tionate volume of average daily production of cement in each month to the average for the year as 100.

(6) The preliminary seasonal factors were then inspected to determine whether they gave evidence in any way of accidental variations of a non-seasonal character. As a test of their accuracy, the provisional figures derived were used to adjust the cement production series for seasonal variations.[1] Occasionally the "adjusted" figures showed uniform increases or decreases between certain successive months for nearly every year. This indicated a fixed error in the seasonal factor in question, attributable usually to the accidental character of the middle items in a small array. The seasonal factor was then altered by an amount sufficient to correct this error. In the case of cement, the first adjustment factor computed for January was too low to be typical, as indicated by a wider range of items above than below the average and by the fact that the provisional "adjusted" figures for January were larger in most of the years than were the corresponding figures for both the preceding December and the subsequent February. Therefore, 60 instead of 59 was adopted as the measure of seasonal variation for January. For both September and August the preliminary seasonal measure was 119, but because the September ratios in the majority of years exceeded those of August, the seasonal factors of 118 for August and 119 for September were finally adopted. Similar cases were found in deriving seasonal factors for practically every product included in the index of physical production. This check, which eliminates constant errors due to accidents of the average of the middle items, is, in a sense,

[1] This was done by dividing the average daily production for any given month by the preliminary measure of seasonal variation for that month.

a partial substitute for the more precise month-to-month comparison provided by the link-relative method.

CHANGING SEASONAL FLUCTUATIONS

One advantage of the ratio-to-moving-average method is that it is easily adaptable for measuring long-time changes in seasonal fluctuations which appear to have occurred in a number of important industries. It seems clear from a study of over 100 statistical series that these shifts in seasonal movements are more typical of industry than fixed and regularly recurring fluctuations.[1] In some lines of manufacture and trade these changes are of such a nature that the month of maximum or minimum activity has actually shifted over a period of years, while in others the change is merely one of an increase or decrease of the percentage of the year's business concentrated in certain months. When the changes are small in magnitude, they may be ignored for purposes of adjustments in an index number, since fixed seasonal factors provide a fair approximation and make unnecessary the more complex and costly computation of changing factors.

In order to determine whether seasonal variations are undergoing real shifts or whether the changes are merely temporary ones caused by transient influences, it is desirable that the data cover a number of years. In most industries for which changing factors were computed by the writers, data were available for about fifteen years, although, in a few series covering only eight years, shifts were sufficiently pronounced to necessitate computation of changing seasonal factors. The process of making these computations is explained here, with illustrations from the data for number of hogs slaughtered under Federal inspection.

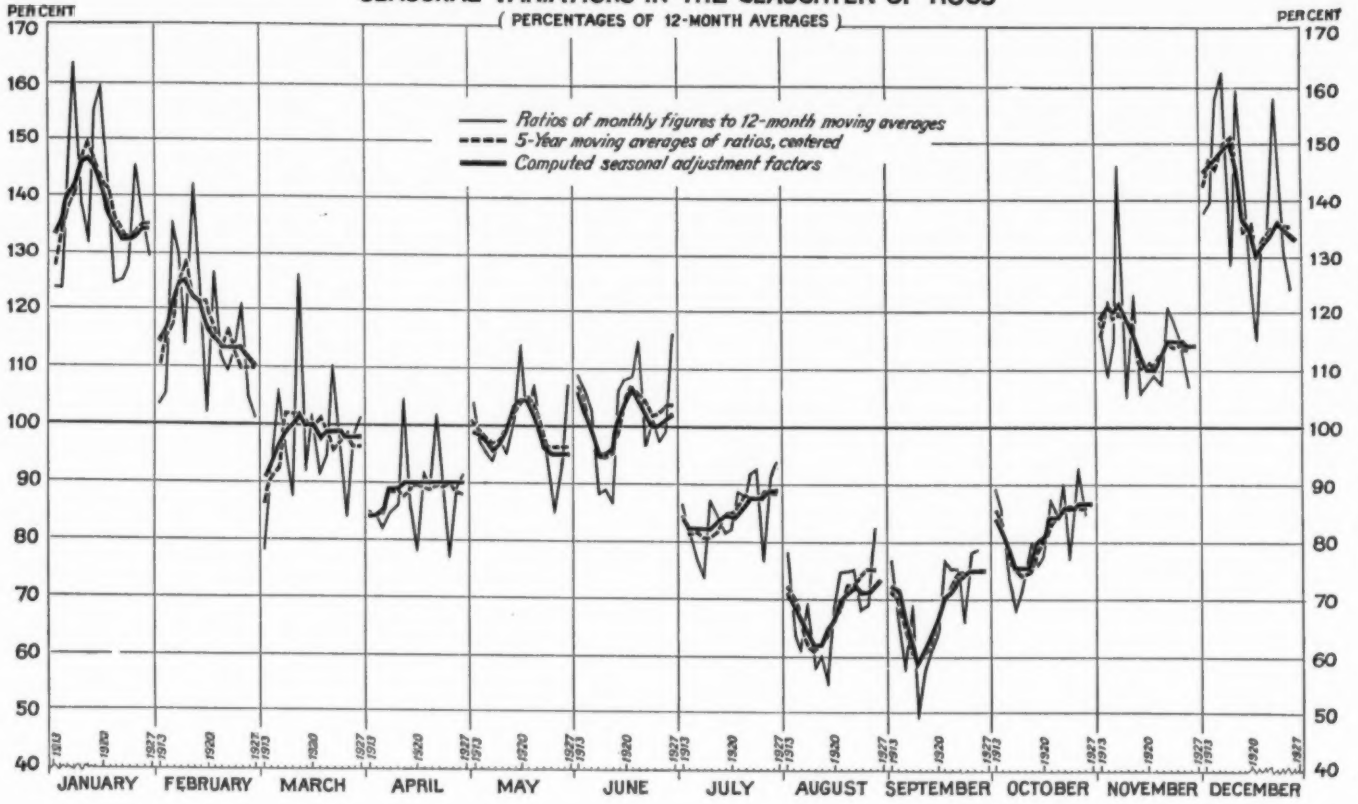
The first two steps described above—the computation of twelve-month moving averages and of the ratios of average daily output to the moving average—were alike in deriving both the fixed and the changing seasonal adjustment factors. In the third step, the ratios for each month were arranged chronologically, instead of in order of magnitude, charted, and inspected for evidences of changes in the seasons. A comparison of the levels of the various monthly sets of curves for number of hogs slaughtered is shown on Chart III. The existence of recurrent seasonal variations is evident. December and January are always high and August and September are always low, but for certain months the comparative level of the ratios has changed over a period of several

[1] Marked changes in seasonal variations were found in cotton and wool consumption, number of hogs, cattle, calves, and sheep slaughtered under Federal inspection, lumber cut, production of wheat flour, automobiles, anthracite, bituminous coal, bee-hive coke, wrapping paper, fine paper, chemical wood pulp, manufactured tobacco and snuff, boots and shoes, and for number of vessels built.

CHART III

SEASONAL VARIATIONS IN THE SLAUGHTER OF HOGS

(PERCENTAGES OF 12-MONTH AVERAGES)



years. In February, July, September, and December, the changes are especially noticeable. The proportionate share of the winter months appears to have declined and that of the early fall months to have increased in the last ten years.

To derive approximate measures of these changes, five-year moving averages of the ratios in each monthly group, arranged chronologically, were computed, with projections for the first and last years on the basis of available data. The broken curve on Chart III represents these averages. In a number of industries extremely large or small ratios caused by some unusual event, such as a strike, were eliminated from the computation.

The curves obtained by the use of these moving averages were then smoothed, free-hand, and the resulting figures were read off the chart. Each yearly group was adjusted to total 1,200, giving consideration to the relationship of each ratio to those for adjoining months of the same year as well as to those for the same month of adjoining years. These adjustments were not made by any fixed mathematical method, but on the basis of inspection and comparison of the ratios and their apparent trends. Thus a set of monthly measures of seasonal variation was obtained for each year. They are illustrated on Chart III by the heavy unbroken lines.

ADJUSTMENTS FOR SPECIAL SEASONAL VARIATIONS

The ratio to moving average has also been adapted to the computation of special Easter seasonal factors for retail sales, which are affected differently in different years by the changing date of Easter within the period from March 22 to April 25. A sliding scale of factors was computed for March and April, based on the average factors derived by the ratio-to-moving-average method for these two months, but giving a greater or smaller share to March or April as the date of Easter was early or late. This process has previously been described in this JOURNAL.[1]

COMPARISON WITH OTHER METHODS

The ratio-to-moving-average method was adopted in preference to one of the other familiar methods [2] of computing seasonal variations for a

[1] No. 160, December, 1927, pp. 493–496.

[2] Descriptions and appraisal of these various methods may be found in the following sources:

Persons, Warren M. Various papers in the *Review of Economic Statistics*, particularly Preliminary Vol. I, and also Chapter X in *Handbook of Mathematical Statistics*, edited by H. L. Rietz, Boston, 1924.
Hart, William L., this JOURNAL, September, 1922, pp. 341–349.
Crum, W. L., this JOURNAL, March, 1923, pp. 607–614.
Hall, Lincoln W., this JOURNAL, June, 1924, pp. 155–166.
Falkner, Helen D., this JOURNAL, June, 1924, pp. 167–179.

number of reasons in addition to those already given. In the first place, the ratios to moving averages contain all of the seasonal element of the months which they represent. The comparison of each month with an annual average provides a more comprehensive measure of seasonal variations than the comparison merely of two adjoining months, as in the commonly-used link-relative method, devised by Professor Persons. Although in most cases there may not be much to choose between the results obtained by the two methods, the moving-average method

TABLE III

SEASONAL FACTORS COMPUTED BY RATIO-TO-MOVING-AVERAGE AND LINK-RELATIVE METHODS*

Month	Monthly seasonal factors			Deviations from assumed monthly factors		Deviations from assumed month-to-month changes	
	Assumed	Link-relative method	Moving-average method	Link-relative method	Moving-average method	Link-relative method	Moving-average method
Jan.	91.9	89.5	91.9	−2.4	....	0.4	0.7
Feb.	91.5	92.5	95.2	+1.0	+3.7	3.4	3.7
March	100.4	98.2	99.0	−2.2	−1.4	3.2	5.7
April	93.6	89.2	90.2	−4.4	−3.4	2.2	2.0
May	95.8	98.3	98.6	+2.5	+2.8	6.9	6.2
June	95.1	97.5	97.3	+2.4	+2.2	0.1	0.5
July	98.3	97.1	95.6	−1.2	−2.7	3.6	4.9
Aug.	103.6	106.7	102.8	+3.1	−0.8	4.3	1.9
Sept.	109.8	110.1	110.6	+0.3	+0.8	2.8	1.6
Oct.	118.0	122.5	120.7	+4.5	+2.7	4.2	1.9
Nov.	105.0	104.2	101.8	−0.8	−3.2	5.3	5.9
Dec.	97.0	94.2	96.3	−2.8	−0.7	2.0	2.7
Average deviation				2.3	2.03	3.2	3.14

* The writers are indebted to Dr. Lorenz for permission to print this table.

should be preferable in a series in which there is a decided trend, broken by sharp cyclical movements.

Dr. M. O. Lorenz, Director of the Bureau of Statistics of the Interstate Commerce Commission, has made an ingenious comparison of these two methods in a hypothetical series with assumed seasonal, secular, cyclical, and fortuitous variations. The results are given in Table III. Both methods missed the assumed seasonal rather badly

Bowley, A. L., and Smith, K. C., *London and Cambridge Economic Service*, Special Memorandum No. 7, July, 1924.

King, Willford I., this JOURNAL, September, 1924, pp. 301–313.

Gressens, O., this JOURNAL, June, 1925, pp. 203–210.

Notes relating to "Meeting on Measurement of Seasonal Variations," this JOURNAL, March, 1925, pp. 426–434.

Mitchell, Wesley C., *Business Cycles, the Problem and its Setting*, New York, 1927, Chapter III, Section III, 4, pp. 233–249.

Carmichael, F. L., this JOURNAL, September, 1927, pp. 339–354.

in certain months, but the deviations from the norm of the figures obtained from the ratio to moving averages were the smaller in seven of the twelve months and averaged less for all the months. In comparing month-to-month changes, as might be expected, the link-relative method provided more accurate results in seven months, but the deviations from the assumed changes for all twelve months averaged slightly less in the case of the moving-average method. The ratio-to-moving-average method seems to be slightly more accurate in this case, but a positive generalization can hardly be drawn from a single example.

The ratio-to-moving-average method requires somewhat more computation than the link-relative method and, because of the use of a twelve-month moving average, it cannot be so readily brought up to date. The ratios derived by this method, on the other hand, provide convenient basic data for analyzing the nature of seasonal variations for the individual months, before determining the final seasonal factors. This makes the method less mechanical and rigid and more adaptable in allowing for accidents of sampling or for fundamental economic changes.

The method based upon monthly means of actual figures is impracticable for production statistics in which pronounced trends and extreme fluctuations caused by cyclical and other influences are usual. The use of ratios to ordinates of trend is an improvement upon monthly means in that it attempts to eliminate the effect of long-time growth, but it is subject to two important criticisms: (1) It makes no attempt to eliminate the influence of the cycle, which in production statistics is frequently dominant, and (2) it depends upon the dubious process of fitting a curve computed from a mathematical formula, generally by the method of least squares, to data that do not fluctuate in accordance with mathematical formulae.

The method devised by Dr. Willford I. King, employing ratios to a free-hand curve representing cyclical fluctuations, furnishes a more logical basis for deriving seasonal factors than the other methods. Some experimentation with this method, however, indicates that the process of drawing free-hand curves is laborious and requires great familiarity with conditions in individual industries. A twelve-month moving average, which, as Dr. Mitchell says, has that "independence of the personal equation" which "gives comfort to many minds," is the nearest approach to an easily computed curve and is fairly representative. The untypical items among the ratios caused by failure of the moving averages to conform exactly to the contour of the cycle may be eliminated, in part, first, by the use only of middle items and, secondly,

by the test of provisional adjustments before definitely adopting a set of seasonal adjustment factors.[1]

In conclusion, it may be said that the ratio-to-moving-average method yields fairly satisfactory results, empirically tested. It cannot be over-emphasized, however, that any process yet devised yields only an approximate measure of seasonal fluctuations and the results must be interpreted with reservations. Although processes of computing measures of seasonal fluctuations may in the future be improved, it seems unlikely that any fixed mathematical method can be devised which may be rigorously applied, without regard to the many diverse elements that influence the movements of industry and trade in ways not capable of expression in terms of mathematical formulae.

[1] Dr. Fred R. Macaulay has devised a mathematically computed curve which is free from seasonal variations and which follows the data much more closely than the moving average, turning promptly with changes in direction of cyclical movements. But computation of this curve is so complicated and laborious and requires data for such long periods of time, that its use is impracticable in a project including a large number of series, many of which cover relatively few years.

THE ARC TANGENT IN TREND DETERMINATION

By F. L. Carmichael,[1] *University of Denver*

During the past twelve years many economic series have undergone what appears to be a permanent change in level. The straight-line tendency commonly experienced prior to the World War was interrupted in many cases in 1915–1917 by a sharp rise, followed in 1920–1921 by an abrupt drop and subsequent resumption of a more regular movement at a level widely different from that of the pre-war period. Changes of this character are evident, for example, in indexes of price change. In series such as stock prices the sharp rise has taken place at a later date, in many cases, with no evidence at present of the subsequent drop. In still others there has been a fairly uniform downward tendency throughout the past fifteen or twenty years. Prices of agricultural and livestock products, deflated by an index of the general price level, may be characterized in this manner.

The inverse trigonometric function known as the arc tangent appears to be adapted to measuring the trend of series behaving as above described. More specifically, its use may be urged when the trend of the series under consideration can be identified with one of the following: (1) A downward tendency approximating the straight-line, but of such nature that projection of a straight line into the future would lead to absurd results, that is, negative or ridiculously small positive values when comparatively large positive values only are possible; (2) approximately a linear growth or decline, followed by an abrupt change in level (rise or drop) and subsequent resumption of the early tendency; (3) approximately a straight-line trend interrupted by a sharp rise or drop, followed by another abrupt change in level and subsequent resumption of the early movement at the same or a different level.

It is desired to explain the characteristics of the arc tangent (the arc or angle whose tangent is a given amount) and to indicate its applicability to the measurement of trend[2] when series behave in the manner outlined.

[1] The writer is greatly indebted to the following: John H. Cover, for valuable criticisms and suggestions; Hiram S. Davis, for suggestions and assistance in the computation; the University of Denver Bureau's fellows in Economic Research, Albert Butler, Emmett Welch and Harry Sukeforth, for assistance in the computation.

[2] While no attempt is made to summarize criteria for curve-fitting, it is desired to emphasize the importance of reasonable projection into the future and the desirability of avoiding gaps in the analysis. Numerous examples are presented in subsequent paragraphs in which the arc tangent and logarithmic

When an angle is assumed to vary from -90 degrees through 0 to $+90$ degrees, its tangent varies from $-\infty$ through 0 to $+\infty$. If x has any value between $-\infty$ and $+\infty$, it follows, therefore, that there is an angle between -90 degrees and $+90$ degrees whose tangent is x. This angle is described as arc tan x. When x is negative (or positive), arc tan x is negative (or positive); for values of x differing only in sign, the corresponding values of arc tan x differ only in sign. With these elementary facts in mind it is easy to visualize from Table I the movement of a curve whose equation takes the form

$$y=a+b \text{ arc tan } x. \tag{1}$$

When x is negative and large numerically, arc tan x changes slowly with x, so that the curve approximates the horizontal. As the origin is approached from the left, the curve rises or falls (depending upon the sign of b) more and more rapidly. The change in level continues as x increases past zero, but with decreasing rapidity; for large *positive* values of x the horizontal is approached again.

TABLE I

VALUES OF THE ARC TANGENT—DEGREES

x	Arc tan x	Arc tan $2x$	Arc tan $5x$	Arc tan $10x$
0.0	0.00	0.00	0.00	0.00
0.2	11.31	21.80	45.00	63.43
0.4	21.80	38.66	63.43	75.96
0.6	30.96	50.20	71.56	80.54
0.8	38.66	57.99	75.96	82.87
1.0	45.00	63.43	78.69	84.29
1.2	50.20	67.38	80.54	85.24
1.4	54.46	70.35	81.87	85.91
1.6	57.99	72.65	82.87	86.42
1.8	60.95	74.48	83.66	86.82
2.0	63.43	75.96	84.29	87.14
2.2	65.56	77.20	84.81	87.39
2.4	67.38	78.23	85.24	87.61
2.6	68.96	79.11	85.60	87.79
2.8	70.35	79.88	85.91	87.96
3.0	71.56	80.54	86.19	88.09

Similarly, curves described by

$$y=a+bx+c \text{ arc tan } x \tag{2}$$

approximate a straight line, with slope b, to the left of (and remote from) the origin, rapidly change level as x increases through zero, and resume the straight-line tendency when x is large. The introduction of a second arc tangent function as in

$$y=a+bx+c \text{ arc tan } x+d \text{ arc tan } (\alpha x+\beta), \tag{3}$$

arc tangent curves appear to possess a decided advantage, so far as these requirements are concerned, over the equation types generally employed. Instead of ignoring the war period in the analysis of time series, as a result of trend difficulties, it is possible by their use (in many cases) to fit one continuous curve to the entire period of the data which will serve as an adequate measure of the trend. Inclusion of the war period in the analysis may be considered of importance also in relation to similar experiences of the future. The method of least squares is used throughout the examples.

α and β being constants arbitrarily chosen, makes possible a second change in level. The term "c arc tan x" may be used to measure a rise, for example, and "d arc tan $(\alpha x+\beta)$" a drop.

When fitting curves of types (1), (2) and (3) by the method of least squares it is necessary to make arbitrary choice,[1] based upon graphical considerations, of the point of origin and of the scale on the x-axis. If a downward tendency approximating the straight-line throughout the period is indicated, the origin is chosen at a point several years prior to the beginning of the data. In the other cases referred to, the origin is taken near a point of rapid change in level. For curve (3) α and β are determined in such manner that $\alpha x+\beta$ will be zero, and hence that "d arc tan $(\alpha x+\beta)$" will change most rapidly, near the second point of abrupt change in level. The effect of changes in scale is apparent from an examination of the various columns of Table I.

Curves of the foregoing types, referred to in subsequent paragraphs as arc tangent curves, have the property (over the periods in which the arc tangent terms are slowly increasing or decreasing) of approximating constant *absolute* changes from year to year. If constant *rates* of change are desired over these periods, the use of

$$\log y=a+bx+c \text{ arc tan } x+d \text{ arc tan } (\alpha x+\beta) \qquad (4)$$

is suggested. This will be referred to as the logarithmic arc tangent.

Prices[2] of Anaconda Copper common stock may be cited as a series illustrating a downward tendency approximating the straight-line. Over the period 1908 through 1926 a straight line, declining slightly less than three dollars a year, fits the data well on the whole, as indicated by a comparison of columns (2) and (3), Table II. Because of its rapid decline, however, and of the fact that it is low in 1926, its projection into the future would be open to serious question. A curve falling with decreasing rapidity is obviously desired. To a degree this requirement is met by the logarithmic straight line, column (4). The 1926 figure, larger than that of the straight line, would appear to be still too small.

[1] The necessity for arbitrary choice of origin and unit on the x-axis is admittedly objectionable. Could it be obviated without loss of flexibility of the curve, the technique would be vastly improved. In logic, however, it appears to fall in the same category as choice of equation type, since both are based upon purely graphical considerations. Similar also is an arbitrary selection of the period used in the *basic* trend computation.

[2] Prices of Anaconda Copper and of other common stocks mentioned in paragraphs following are tenth-of-month prices, obtained by averaging the high and low figures for the five days Monday to Friday, inclusive, of the week beginning the seventh of the month and adjusted, where necessary, for artificial price changes resulting from stock dividends, changes in par values, reorganizations, and issuance of stock rights. Adjustments were made in all cases to the January, 1927, level, using prices for this month as compiled and raising or lowering others as required. Annual averages of monthly prices are considered June twenty-fifth figures.

TABLE II

ANACONDA COPPER COMMON STOCK PRICES

Year	Actual data (annual averages of tenth-of-month prices)	Average tendencies derived from equations of the form							
		$y = a + bx$	$\log y = a + bx$	$y = a + b$ arc tan x, with					
				Origin June 25, 1900, and yearly changes in x			Origin June 25, 1892, and yearly changes in x		
				1.0	0.9	0.75	1.0	0.25	0.125
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1908	$83.57	$91.02	$95.40	$99.25	$99.51	$99.51	$96.57	$96.47	$96.03
1909	95.22	88.14	91.04	91.21	91.43	91.45	90.91	90.93	90.80
1910	86.42	85.27	86.89	84.77	84.93	84.97	85.89	85.97	86.06
1911	75.04	82.40	82.92	79.48	79.61	79.65	81.39	81.51	81.72
1912	82.64	79.53	79.13	75.08	75.17	75.20	77.31	77.48	77.77
1913	72.14	76.66	75.52	71.34	71.40	71.43	73.65	73.81	74.14
1914	66.83	73.79	72.07	68.14	68.17	68.19	70.33	70.47	70.80
1915	68.05	70.92	68.78	65.35	65.36	65.39	67.28	67.41	67.72
1916	87.85	68.05	65.64	62.91	62.91	62.93	64.48	64.60	64.86
1917	74.19	65.18	62.64	60.76	60.74	60.76	61.92	62.00	62.22
1918	65.78	62.31	59.78	58.86	58.81	58.82	59.54	59.61	59.76
1919	65.23	59.44	57.05	57.15	57.09	57.09	57.33	57.38	57.46
1920	54.69	56.57	54.44	55.60	55.53	55.53	55.28	55.31	55.32
1921	39.53	53.70	51.96	54.21	54.13	54.12	53.39	53.38	53.31
1922	51.47	50.83	49.59	52.95	52.85	52.84	51.61	51.59	51.43
1923	43.61	47.96	47.32	51.80	51.68	51.67	49.97	49.89	49.66
1924	35.44	45.09	45.16	50.73	50.61	50.60	48.40	48.31	48.00
1925	42.96	42.22	43.10	49.76	49.63	49.61	46.95	46.82	46.43
1926	47.78	39.35	41.13	48.86	48.72	48.70	45.57	45.42	44.95
Limiting value		$-\infty$	0.00	26.30	25.95	25.84	+0.13	−1.34	−5.81

The arc tangent curve of form (1) above, with suitable choice of the origin and of the unit on the x-axis, appears more nearly to meet the requirements. It approximates a straight line throughout the period of the data; and its projection for a few years into the future is far more dependable.

To illustrate the effects produced by changes in the point of origin and in the x-scale, the results for six cases are presented. Columns (5), (6), and (7) differ only so far as the x-unit is concerned; similarly, columns (8), (9), and (10). Decreasing the yearly change in x lowers the limiting value. By a comparison of columns (5) and (8), for which the units on the x-axis are identical, it may be seen that moving the point of origin to the left causes the curve to approximate a straight line more closely and hence to have a lower limiting value.[1]

[1] Equations of the curves fitted to Anaconda Copper common stock prices, period 1908 through 1926, are:

(1) $y = 65.18 - 2.8705x$, origin at June 25, 1917, yearly change in x, 1;
(2) $\log y = 1.79685 - .02030x$, origin at June 25, 1917, yearly change in x, 1;
(3) $y = 947.77 - 10.2385$ arc tan x, origin at June 25, 1900, yearly change in x, 1;
(4) $y = 863.27 - 9.3035$ arc tan x, origin at June 25, 1900, yearly change in x, 0.9;
(5) $y = 726.60 - 7.7863$ arc tan x, origin at June 25, 1900, yearly change in x, 0.75;
(6) $y = 2427.42 - 26.9699$ arc tan x, origin at June 25, 1892, yearly change in x, 1;
(7) $y = 625.86 - 6.9689$ arc tan x, origin at June 25, 1892, yearly change in x, 0.25;
(8) $y = 339.22 - 3.8337$ arc tan x, origin at June 25, 1892, yearly change in x, 0.125.

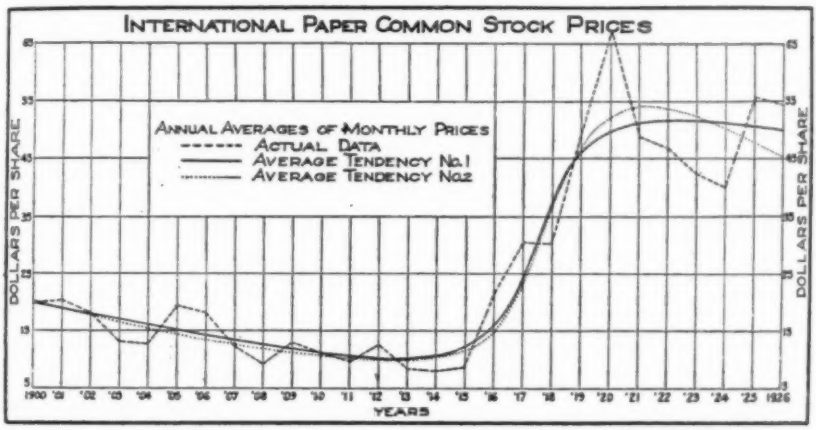
All arcs should be expressed in degrees; one dollar is the unit on the y-axis. The yearly changes in x are indicated for all arc tangent curves as used in the least squares computations.

The conditions here outlined are readily identified with the corresponding columns of Table II.

Prices of International Paper common stock graphed in Chart I, show a marked and rather consistent decline from 1900 through 1914, followed by a rapid rise toward 1920 and a subsequent drop. Arc tangent and logarithmic arc tangent curves were fitted to the data as indicated in Table III. The arc tangent curve (average tendency No. 1 of the chart, column 3 of the table) fits the data well with the possible exception of the past few years. Since it approximates constant *absolute* changes at points remote from the origin, the portion of the curve past 1923 is practically parallel to that prior to 1910. Assuming that it is not falling as rapidly as it should, the logarithmic arc tangent

CHART I

ANNUAL PRICES OF INTERNATIONAL PAPER COMMON STOCK AND ARC TANGENT AND LOGARITHMIC ARC TANGENT CURVES, NOS. 1 AND 2, RESPECTIVELY, FITTED BY THE METHOD OF LEAST SQUARES



is suggested, due to the property it enjoys of changing its level, at points where the arc tangent terms are slowly changing, at approximately a constant *rate*. Three cases are shown in the table, columns (4), (5), and (6); the last is charted as average tendency No. 2. If the foregoing assumption is valid, No. 2 is obviously preferable to No. 1.

For columns (3) and (4), the origin and x-scale are identical; during the period 1914–1921 the arc tangent rises earlier than the logarithmic arc tangent, an important difference in equation type to be borne in mind when choosing the point of origin. In column (5) the origin is moved six months to the left, thus bringing the curve nearer the data during the crucial period. Note the more rapid rise evident from column (5) as compared with column (3). By decreasing the yearly change in x from 0.75 to 0.6, the only difference between the conditions of

TABLE III

INTERNATIONAL PAPER COMMON STOCK PRICES

Year	Actual data (annual averages of tenth-of-month prices)	Average tendencies derived from equations of the form			
		$y=a+bx+c$ arc tan x (origin Dec. 25, 1917; yearly change in x, 0.75)	$\log y=a+bx+c$ arc tan x (origin Dec. 25, 1917; yearly change in x, 0.75)	$\log y=a+bx+c$ arc tan x (origin June 25, 1917; yearly change in x, 0.75)	$\log y=a+bx+c$ arc tan x (origin June 25, 1917; yearly change in x, 0.6)
(1)	(2)	(3)	(4)	(5)	(6)
1900	$19.83	$19.91	$19.25	$19.82	$20.39
1901	20.33	18.94	18.22	18.60	18.99
1902	18.07	17.98	17.24	17.45	17.70
1903	13.09	17.04	16.33	16.39	16.52
1904	12.73	16.10	15.48	15.40	15.43
1905	19.51	15.20	14.68	14.49	14.43
1906	18.14	14.32	13.95	13.65	13.52
1907	12.18	13.47	13.26	12.88	12.71
1908	9.24	12.67	12.64	12.18	11.97
1909	13.01	11.93	12.08	11.56	11.33
1910	11.11	11.27	11.58	11.01	10.79
1911	9.70	10.73	11.16	10.57	10.36
1912	12.55	10.37	10.84	10.24	10.08
1913	8.48	10.30	10.66	10.10	10.02
1914	8.03	10.75	10.71	10.27	10.36
1915	8.78	12.19	11.23	11.16	11.52
1916	21.70	15.82	12.92	14.10	14.82
1917	30.75	24.40	18.36	23.35	23.29
1918	30.38	37.58	31.76	38.67	36.60
1919	49.43	46.15	45.13	48.85	47.07
1920	67.01	49.79	51.96	53.06	52.36
1921	48.92	51.23	54.47	54.20	54.11
1922	46.89	51.67	54.74	53.23	53.81
1923	42.42	51.60	53.83	51.59	52.37
1924	40.07	51.25	52.28	49.50	50.29
1925	55.81	50.71	50.38	47.17	47.87
1926	54.75	50.05	48.30	44.75	45.30
1927	....	49.30	46.14	42.34	42.69
1928	....	48.50	43.97	39.95	40.11

columns (5) and (6), the rapidity of the rise is decreased and the curve brought into closer proximity with the arc tangent.[1]

To clarify the procedure leading to the choice of the yearly change in x, average tendency No. 1 may be taken as an illustrative example. By reference to the chart of actual data it is apparent that the midpoint of the period of rapid rise is approximately midway between the 1917 and 1918 points. December 25, 1917, is therefore chosen as the origin. Ordinates of the average tendency are estimated at this point and at pairs of points ½, 1, 1½, and 2 years away (in this case), from which rises to the right and declines to the left of the origin are determined. Corresponding rises and declines are averaged to obtain the

[1] Equations of International Paper common stock price curves, fitted to the period 1900 through 1926, are:

(1) $y=30.99-1.4170x+.3465$ arc tan x, origin at December 25, 1917, yearly change in x, 0.75;
(2) $\log y=1.38295-.034338x+.0064119$ arc tan x, origin at December 25, 1917, yearly change in x, 0.75;
(3) $\log y=1.36826-.039728x+.0067518$ arc tan x, origin at June 25, 1917, yearly change in x, 075;
(4) $\log y=1.36718-.055718x+.0074183$ arc tan x, origin at June 25, 1917, yearly change in x, 0.6.

All arcs should be expressed in degrees; one dollar is the unit on the y-axis. See Table III.

figures shown in column (2) of Table IV,[1] the last of which, $17.75, is the base for column (3). Yearly changes in x are now assumed and the corresponding values (at half-yearly intervals) of x and of arc tan x determined. The latter expressed as percentages of the corresponding number of degrees shown in the last line, columns (6), (9), and (12), are now compared with column (3).

TABLE IV

COMPARISON OF CHANGES IN ARC TANGENT x FOR ASSUMED YEARLY CHANGES IN x AND OF ESTIMATED CHANGES IN THE AVERAGE TENDENCY

(Average tendency No. 1 of International Paper common stock prices, with origin at December 25, 1917)

No. of years removed from Dec. 25, 1917	Average of estimated changes in level from Dec. 25, 1917		Values of x and of arc tan x for assumed yearly changes in x								
			Yearly change in x, 1			Yearly change in x, 0.75			Yearly change in x, 0.5		
			x	Arc tan x		x	Arc tan x		x	Arc tan x	
	Actual	Per cent*		Degrees	Per cent*		Degrees	Per cent*		Degrees	Per cent*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
½	$6.50	37	0.5	26.57	42	0.375	20.56	37	0.25	14.04	31
1	11.25	63	1.0	45.00	71	0.750	36.87	65	0.50	26.57	59
1½	15.25	86	1.5	56.31	89	1.125	48.37	86	0.75	36.87	82
2	17.75	100	2.0	63.43	100	1.500	56.31	100	1.00	45.00	100

* Actual data in last line of table = base.

The close agreement of (9) with (3) shows that a yearly change in x of 0.75 is appropriate. (Since arc tan $x = 0$ at the origin, the values of arc tan x at other points are proportional to the changes in level from December 25, 1917, produced by the presence of this function in the equation. The changes at equal time-intervals to the right and the left are the same numerically, but opposite in sign.)

Determination of the yearly change in x in the case of the logarithmic arc tangent curve may be made similarly by use of a graph of the logarithms of the actual data on simple arithmetic paper, or of the actual data on semi-logarithmic paper. If the latter is employed, it is important to note that distances along the vertical are to be compared and not the actual changes in original data units.

Prices of American Car and Foundry common stock are shown in Table V [2] as a series evidencing a fairly uniform growth over most of

[1] A little acquaintance with arc tangent curves enables one to choose an appropriate yearly change in x in the case of many series through an examination of a graph of the actual data only, thus obviating the necessity of making the computation outlined in Table IV. By proceeding as here explained, it is thought that the inexperienced will find but little difficulty in the derivation of arc tangent trend curves.

[2] Equations for the data of Table V, period 1900 through 1926 in all cases, are as follows:

(1) American Car and Foundry common stock prices: $y = 60.74 + 2.134x + .3467$ arc tan x, origin at December 25, 1919, yearly change in x, 0.5, y-unit, $1;

(2) Central Leather common stock prices: $y = 24.47 - 1.7613x + .5191$ arc tan $x - .4193$ arc tan $(x - 4.5)$, origin at December 25, 1915, yearly change in x, 1, y-unit, $1;

(3) Annual index of wholesale prices in the United States: $y = 121.72 + .6961x + .7236$ arc tan $x - .4552$ arc tan $(3x - 16.5)$, origin at May 1, 1917, yearly change in x, 1.5, y-unit, 1 per cent.

All arcs should be expressed in degrees.

the period, but with an abrupt rise during the last few years. Note that the rapid rise of the arc tangent curve, column (3), is spent by 1926 and that its projection into the future is not unreasonable.

In the prices of Central Leather common stock the characteristics of International Paper are duplicated except that the gradual decline of the past few years is replaced by a sharp drop. In order that the arc tangent curve may fit such a series it is necessary that a second arc tangent function be introduced into the equation. Graphically, the rise and the drop appear to be almost equally rapid in point of time, so that the value of α in equation (3) was chosen as 1. The point of most rapid rise was estimated to be December 25, 1915; that of most rapid drop, June 25, 1920. From the one point to the other is a period of four-and-one-half years. The former was taken as the origin, the latter as the point where the term "d arc tan $(\alpha x+\beta)$" should change most rapidly and hence $\alpha x+\beta$ should equal zero. By examining a graph of the actual data and considering the characteristics of the arc tangent function, a suitable yearly change in x was estimated to be 1. It follows, therefore, that the necessary value of β is -4.5.

TABLE V

AMERICAN CAR AND FOUNDRY AND CENTRAL LEATHER COMMON STOCK PRICES AND THE UNITED STATES BUREAU OF LABOR STATISTICS INDEX OF WHOLESALE PRICES

Year	American Car and Foundry		Central Leather		Index of wholesale prices	
	Actual data*	Average tendency	Actual data*	Average tendency	Actual data (per cent)	Average tendency (per cent)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1900	$10 16	$10.77	$40.97	$43.50	80.5	81.35
1901	13.34	11.94	42.94	41.81	79.3	82.48
1902	16.07	13.12	42.31	40.13	84.4	83.62
1903	16.53	14.33	33.57	38.46	85.5	84.78
1904	10.55	15.55	29.29	36.82	85.6	85.97
1905	18.30	16.79	43.37	35.21	86.2	87.16
1906	20.86	18.05	40.93	33.62	88.6	88.40
1907	19.07	19.35	24.52	32.09	93.5	89.67
1908	18.46	20.69	24.27	30.64	90.1	90.99
1909	29.99	22.07	36.24	29.30	96.9	92.39
1910	27.94	23.52	37.40	28.13	100.9	93.91
1911	26.44	25.06	26.83	27.26	93.0	95.58
1912	28.72	26.71	25.68	26.94	99.1	97.52
1913	23.50	28.53	24.06	27.80	100.0	99.93
1914	25.03	30.60	33.40	31.65	98.1	103.30
1915	29.99	33.04	42.91	44.55	100.8	109.05
1916	32.65	36.11	65.55	69.23	126.8	123.38
1917	34.13	40.27	83.67	81.06	177.2	171.37
1918	40.31	46.36	66.59	82.06	194.3	205.20
1919	56.22	55.34	88.58	75.61	206.4	213.86
1920	67.56	66.14	64.79	56.77	226.2	211.60
1921	63.76	75.12	34.10	37.28	146.9	154.56
1922	84.19	81.20	36.99	28.60	148.8	149.79
1923	84.91	85.36	24.52	24.03	153.7	150.16
1924	83.01	88.43	14.36	20.88	149.7	151.16
1925	105.10	90.88	18.70	18.34	158.7	152.31
1926	101.47	92.95	11.35	16.09	151.0	153.48
1927	.....	94.77	.....	14.02	.....	154.67
1928	.....	96.42	.....	12.05	.....	155.84

*Annual averages of tenth-of-month prices.

In the case of the Bureau of Labor Statistics index of wholesale prices the decline in 1921 was far more abrupt than the preceding rise. May 1, 1917, where the index appeared to increase most rapidly, was chosen as the origin; the yearly change in x was taken as 1.5. With this choice of the x-scale in mind, it was necessary to select a value of α (by consideration of the characteristics of the arc tangent function) which would insure the desired rapid drop. Estimating this to be 3 and noting that the sharpest drop seemed to occur at January 1, 1921, the value of β satisfying the condition that $\alpha x+\beta$ shall equal zero at this point was found to be -16.5.

The choice of the yearly change in x, 1.5, may be based upon a computation similar to that explained in a foregoing paragraph for International Paper common stock prices. The steps leading to the determination of α and β may be detailed somewhat analogously as follows.

Ordinates of the average tendency at January 1, 1921, and at pairs of points 1/10, 3/10, 5/10, and 7/10 of a year away (in this case) are estimated, from which declines to the right and rises to the left are determined. Corresponding rises and declines are averaged to obtain the figures shown in column (2) of Table VI, the last of which is the base for column (3). Since $\alpha x+\beta$ must equal zero at January 1, 1921, it follows that its numerical value at any point to the right or the left may be determined by multiplying the number of years removed therefrom by $1.5\,\alpha$. Three values of α, namely, 2, 3, and 4, are assumed and the cor-

TABLE VI

COMPARISON OF CHANGES IN ARC TANGENT ($\alpha x+\beta$) FOR ASSUMED VALUES OF α AND OF ESTIMATED CHANGES IN THE AVERAGE TENDENCY

(United States Bureau of Labor Statistics index of wholesale prices, yearly change in x, 1.5, with $\alpha x+\beta=0$ at the point of most rapid decline, *i. e.*, January 1, 1921)

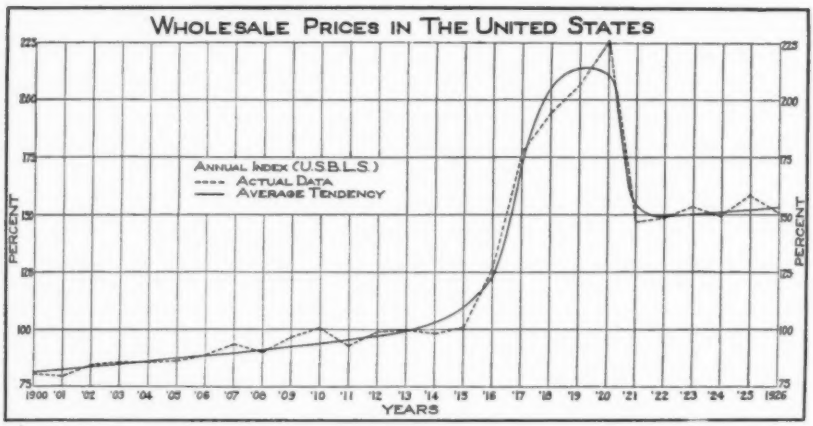
No. of years removed from Jan. 1, 1921	Average of estimated changes in level from Jan. 1, 1921		Values of $(\alpha x+\beta)$ and of arc tan $(\alpha x+\beta)$ for assumed values of α								
			Value of α, 2			Value of α, 3			Value of α, 4		
			$(\alpha x+\beta)$	Arc tan $(\alpha x+\beta)$		$(\alpha x+\beta)$	Arc tan $(\alpha x+\beta)$		$(\alpha x+\beta)$	Arc tan $(\alpha x+\beta)$	
	Actual	Per cent *		Degrees	Per cent *		Degrees	Per cent *		Degrees	Per cent *
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1/10	10.0	33	0.3	16.70	26	0.45	24.23	33	0.6	30.96	40
3/10	23.0	75	0.9	41.99	65	1.35	53.47	74	1.8	60.95	80
5/10	27.5	90	1.5	56.31	87	2.25	66.04	91	3.0	71.56	93
7/10	30.5	100	2.1	64.54	100	3.15	72.29	100	4.2	76.61	100

* Actual data in last line of table = base.

responding values of $\alpha x+\beta$ and of arc tan $(\alpha x+\beta)$ determined. The close agreement of column (9) with column (3) shows that 3 is an appropriate value of α. From the origin, May 1, 1917, to January 1, 1921, is a period of $3\frac{2}{3}$ years, so that x equals 1.5 $(3\frac{2}{3})$ or 5.5 at the latter date. But $\alpha x+\beta=3(5.5)+\beta=0$ at this point, and hence β must equal -16.5.

It is interesting to note that the curve fitted to the index of wholesale prices enjoys the property during the period 1900 through 1926 of having not more than two successive (annual) points of the actual data on the same side of the curve. This is shown graphically in Chart II.

CHART II
ANNUAL INDEX OF WHOLESALE PRICES IN THE UNITED STATES AND ARC TANGENT CURVE FITTED BY THE METHOD OF LEAST SQUARES



In the foregoing examples it may be seen that all arc tangent and logarithmic arc tangent curves at and beyond the last point of the data change level but slowly. The flexibility of these curves which makes possible a reasonable projection into the future, following a sharp rise or drop near the end of the series, is considered an advantage not possessed by equation types commonly employed in trend determination. Moreover, it is possible adequately to measure trend by one arc tangent or logarithmic arc tangent curve in many cases requiring, by the use of other equation types, separate computations for each of two or more periods, with perhaps gaps in the analysis.

THE STATISTICS OF ROOM CONGESTION

PURPOSE AND TECHNIQUE

By Edith Elmer Wood, *Chairman, Committee on Housing, American Association of University Women*

The number of rooms per family is one of the suggested new inquiries[1] which is being seriously considered for inclusion in the 1930 Census by the Director of the Census and the Joint Advisory Committee on the Census of the American Statistical Association and American Economic Association. Partly for the purpose of affording an American try-out which would be of service in a national enumeration, and partly, of course, for the local benefits to be obtained from the local inquiry, the Board of Education of Detroit has included the number of rooms per family and the number of persons per family, on a city-wide basis (*i. e.*, not confined to families with children of school age), in its 1928 School Census. The Detroit Department of Health has contributed to the cost of tabulation, and the Michigan Housing Association is completing the work and will publish the results.

In view of these evidences of an American awakening to the importance of the subject, it may not be out of place to offer a brief summary of the extensive European experience with this inquiry, (1) as to the practical uses to which these figures may be put, and (2) as to the statistical methods employed in securing and presenting them.

The principal object of collecting information as to the number of rooms per family is to obtain a coefficient for the density under which various sections of the population live, in order to have a scientific basis for health and housing programs. The coefficient of density may be expressed either as "persons per room" (the number of individuals composing the family divided by the number of rooms they inhabit), or as the reciprocal, "rooms per person." The former has been the usual practice. The French Census of 1901 and 1911 and the British Census of 1921 follow the inverse order. Making due allowance for the difficulty of adjustment to an unfamiliar method, it seems to the writer that the more customary procedure (persons per room) calls up a clearer picture to the mental vision at the higher densities, which are the points of social significance.

Room density, when greater than one person per room, has been

[1] First proposed in the United States on a national scale by the American Association of University Women in 1925, and since endorsed by a considerable number of other organizations and individuals.

proved to have an important relation to the general death-rate, infant death-rate, and the prevalence of communicable diseases. The chances of infection increase with the multiplication of contacts. In homes where a sick person cannot have a room to himself, frequently not even a bed to himself, the probability that every germ disease, from a cold in the head to infantile paralysis, will "go through the family" is greatly heightened.

The effect of over-crowding on the nervous system is known to be marked, but has not been carefully studied. Solitude and quiet are impossible of attainment. Babies cry, children quarrel, parents are irritable, nerves are on edge, tempers explosive. There is no peaceful spot where babies and young children can take day-time naps or sleep in the early evening hours. School children have no quiet place in which to study. Mothers, above all, who have no respite during the whole twenty-four hours, suffer from nervous and mental breakdowns.

The connection between extreme over-crowding and sexual immorality, including incest, is well known. Where privacy is impossible, and where old and young of both sexes are herded together like animals, the inhibitions of civilization melt away.

Over-crowding also connects with other social problems. The appeal of the nearest bootlegger's as a refuge from the discomforts of home often proves irresistible to the father of the family. Adolescent boys and girls escape first to the street and later to the dance halls and pool rooms. Alcoholism, desertion and juvenile delinquency come next, and the disintegration of the family is complete.

The relation between over-crowding and the major crime wave has yet to be seriously studied, but it is a lead worth following.

Just what constitutes "crowding" and "over-crowding," is a matter of local definition, which varies considerably.[1] The best usage would seem to favor "more than one person per room" or "more persons than rooms" as the definition of crowding, and "more than two persons per room" of over-crowding. A few countries, where room congestion is very prevalent, place the limit of illegal over-crowding at "more than three persons per room," or even at "more than four persons per room." The most elaborate classification is the Norwegian, which distinguishes five room densities, as follows: (1) Very spacious. More than two rooms in excess of the number of persons. (2) Spacious. One room per person, or one or two rooms in excess. (3) Crowded. More than one, but not more than two persons per room. (4) Over-

[1] The minimum airspace per adult in bedrooms (400, 500 or 600 cubic feet) found in the health or housing code of a number of American cities and a few entire states, is not intended for, or at all adapted to be used as a statistical measure of room congestion.

crowded. More than two, but not more than three persons per room. (5) Greatly over-crowded. More than three persons per room.[1]

Whatever their definition of over-crowding may be, nearly all European countries know from their census how much they have of it and where it is located. Among the countries which have based national housing policies upon this information are Great Britain, France, Germany, Belgium, Holland, Switzerland, Italy, Norway, Sweden, Denmark, Finland, Austria, Hungary, Czecho-Slovakia and Poland.

What knowledge we have of over-crowding in American families is derived from studies by sample—and often by very small sample—made sometimes by public bureaus, more often by private organizations or individuals with widely varying standards and degrees of competence. How accurate such studies are is usually open to question. How typical the samples are, is even more doubtful. According to the recent pamphlet of the International Labour Office on the "Methods Employed in the Statistics of Housing,"[2] this is the way in which all countries started, but most European nations passed beyond it before the end of the nineteenth century. The United States and Japan are the most conspicuous examples still lingering in the study-by-sample stage.

Our National Census does, it is true, give us the number of dwellings and the number of families inhabiting them. The dwellings which it gives, however, are not family-units—the *logements*, *Wohnungen*, etc., of continental Europe or the "tenements" of the British Census. They are the *batiments d'habitation*, or residential buildings. A great apartment house accommodating hundreds of families counts but one dwelling under our system, which, with the increase in apartment houses, is coming to have less and less significance.[3]

One set of housing figures we do, indeed, have on a national scale, which other nations lack—the statistics of home ownership, encumbered and unencumbered.[4] These are of distinct economic and social value. But we are the only nation which has preferred them to the congestion figures which are so closely related to health and decency.

Our 1920 Federal Census contained two items on its farm schedule directly related to housing, water piped into the house and electric light or gas.[5] Admitting the fairly fundamental importance of the

[1] See p. 34, Vol. VII, Housing Statistics—Cities, Census of Norway, December 1, 1920 (*Folketellingen i Norge*, 1 desember 1920, Syvende hefte, Boligstatistikk-Byer).

[2] *Les Méthodes de la Statistique de l'Habitation*, Bureau International du Travail, Geneva, 1928, pp. 57–59.

[3] *Fourteenth Census of the United States*, "Population, Dwellings and Families," p. 3.

[4] *Ibid.*, Vol. II, Chapter XV, "Ownership of Homes."

[5] *Ibid.*, "Agriculture, Selected Farm Expenses, Coöperation, and Farm Facilities," Vol. V, Chapter VIII, pp. 512–514.

water query, students of housing must regret deeply the wasted opportunity involved in devoting the other inquiry to a point of such secondary—or even tertiary significance as method of lighting. One wonders at the psychology behind the choice.

Previous to the Detroit inquiry of the current year, the only city-wide room-density figures in the United States were those contained in the Tenement House Census of Boston made by the Massachusetts Bureau of Statistics of Labor in 1891. Studies by sample, as already stated, have been numerous. Hardly a housing survey has been made anywhere which does not at least dabble in the subject—sometimes only by way of citing individual cases of bad over-crowding. Probably the most valuable studies have been those made by the Philadelphia Housing Association, which deal with some thousands of families and are repeated from year to year.[1] The study of "Living Conditions for Small-wage Earners in Chicago," made by Elizabeth A. Hughes for the Department of Public Welfare in 1925, gives persons-per-room figures for 1526 families. The New York State Commission of Housing, now replaced by the State Board of Housing, has given from time to time, surveys of the same eight blocks, intended to be typical, which were first surveyed in 1920 to show the extent of congestion due to the post-war housing shortage, and more recently, the extent to which it has been relieved by building. Because of their periodic and official character, these block surveys have had considerable importance. But it is always open to anyone who objects to the conclusions drawn to claim that the eight blocks chosen in various parts of the city were exceptional.

The next step after securing room density figures is the attempt to correlate them with other social measurements. The Children's Bureau of the Department of Labor, in its infant mortality studies, has always considered room congestion as an important factor. The earlier reports were suggestive rather than scientifically convincing. Infant death-rates in crowded homes were strikingly high, but the number of cases was small, and there was no means of judging how much poverty and race had counted for in the picture. In the Baltimore study of 1923, this objection was successfully met. The numbers were sufficiently large to permit classification by race and by income as well as by density coefficient. When in each racial group of each economic sub-division, the rate of infant mortality is shown to rise sharply with increased room density, we have a result of great value. The over-crowding factor has been very nearly isolated as a cause and its effect on infant mortality measured.

[1] For additional examples, see the author's *Housing of the Unskilled Wage Earner*, p. 135.

A study of the relation between room density and sickness and death-rates was made in 1919 and 1920 by the Health Commissioner of Detroit, Dr. Henry F. Vaughan, in 42 selected city blocks.[1] General death-rate, infant death-rate, tuberculosis case- and death-rates, and the case- and death-rates of influenza and pneumonia were shown to be highest on the blocks with greatest room density.

Reverting to Europe, one of the most recent contributions[2] is a Glasgow study of the incidence of tuberculosis cases, in the Eastern District of the city, from 1910 to 1924, in relation to the number of rooms occupied by the family. Cases are plotted by age and sex, but for both sexes and all ages, the rate of incidence is much higher in the one-room homes than in those with two rooms, and in two-room homes than in those with three. The record goes no farther, but in that district only 5 per cent of households occupy more than three rooms. Racially, these Glasgow families are no doubt homogeneous to a high degree, but one would like to see the income item eliminated and room density substituted for the absolute number of rooms.

Professor A. L. Bowley of London, in June, 1923, read before the Royal Statistical Society a paper entitled "Death-rates, Density, Population, and Housing." It is an elaborate and highly technical study of the correlation between general and infant death-rates and four possible measures of congestion in living conditions, (1) density of population (persons per acre), (2) room density (persons per room), (3) crowding (per cent of population living more than one person to a room), and (4) over-crowding (per cent of population living more than two persons to a room). The coefficient of correlation was found to be highest in the case of crowding and lowest in that of density.

Among earlier European reports which have been fruitful of results, although not always eliminating all other possible contributory causes, have been Glasgow and Berlin studies of comparative death-rates in one-room, two-room, three-room and four-or-more-room homes. Well known also are the Glasgow studies of the physical development (height and weight) of children in homes containing various numbers of rooms.

If we had room density figures in the United States, we might be able to answer the question why the infant mortality rate in certain New England mill towns is twice as high as the average for the country, or why the rural death-rate for Negroes in the State of Georgia is only 9.9, while that in the cities is 23. We should certainly know much

[1] See *Proceedings of the Eighth National Conference on Housing*, 1920, pp. 183–198, and *The Survey*, January 8, 1921.

[2] See *Garden Cities and Town Planning*, February, 1928.

better than from block studies whether over-crowding in New York justifies the continuation of rent restriction. We should know how many Negro families in the South, how many Mexican families in the Southwest, how many mountaineer families of Anglo-Saxon origin, are living under the sub-normal standards found in one- and two-room cabins. We could learn the extent to which the effort of the public schools is frustrated by bad conditions in children's homes. And, as already suggested, we might be able to throw light on the origin of the crime wave.

Turning now from the content and uses of room density statistics to the methods employed in collecting them, it will be observed that no great technical difficulties are involved. Four statistical units are used, which must, of course, be carefully defined. These are (1) the family, (2) the dwelling inhabited by the family, (3) the individual, and (4) the room.

(1) Obviously, the family to be enumerated is the so-called census family (*ménage*), the whole group of persons who are inmates of a given dwelling (*logement*), whether related to each other by blood or not. It includes servants, boarders and lodgers. Whether it includes those who habitually live there, even if actually absent, and excludes casuals who happen to be found there (as is the American Census practice and also that of France), or whether the count is made of those actually present on a certain date (as is the British practice), is a matter of local custom.

(2) The dwelling with which we are concerned (*logement, Wohnung*) is, of course, the family home, whether a whole house, an apartment or a single room. It must not be confused with the dwelling of our American Census. Probably we should seek a new term in one case or the other. The treatment of non-family dwellings—hotels, schools, asylums, etc.—has not been standardized. The British method of leaving them wholly out of account in recording rooms per family would seem to be the most satisfactory. They are an element of inaccuracy when included.

(3) The only element of doubt in regard to the individual lies in the matter of age. The general practice has been to disregard it. But in some places children below twelve have been counted as one-half, and those below five as one-quarter, or in a few cases disregarded altogether. Other proportions, such as one-third and two-thirds, and other dividing lines as to age are occasionally found.

(4) The definition of the room which is to be counted and the lists of what do and what do not come under the definition have shown a wide variation which has not entirely disappeared. One of the earliest

important inquiries was that of 1864 in Berlin, where the rooms to be counted were those that could be *heated*.[1] This curious distinction long persisted in German, Swiss and Scandinavian statistics, but is now largely discarded. The German Census of 1927 discarded it completely. In southern countries, it would obviously be meaningless, and even in the northern ones it became increasingly clear that the distinction to be made was whether a room was, in fact, lived in, to which the presence or absence of heating facilities was by no means always a sure clue.

The inclusion or non-inclusion of the kitchen[2] in the count has also been a bone of contention. It is still counted separately in some countries. As the kitchen is usually, in working class families, not only a place where food is prepared, but the family dining-room and sitting-room, and sometimes a bedroom as well, it has become increasingly the custom to count it with the other rooms. This has always been the practice in Great Britain.

Some countries for a time rather curiously omitted servants' rooms and attic or mansard rooms,[3] while counting the persons who slept in them. This was obviously misleading and has virtually passed out.

The French and some others define a room as a space enclosed by walls and partitions reaching to the ceiling and having a specified minimum size. The French requirement is that it shall be large enough to contain a single bed for an adult.[4] The city of Amsterdam defines a room as an enclosure with a minimum area of four square meters.[5]

The Norwegians have an elaborate classification of rooms: (1) kitchens, (2) dwelling rooms of common size which may be used all the year, (3) servants' rooms or other small rooms used all the year, (4) rooms used in summer only.[6] But they add them all together when it comes to calculating room density. The Norwegian Census goes into housing conditions with extraordinary thoroughness. Volume VII of the Census of 1920 is devoted to the statistics of housing in cities and Volume VIII in rural districts. Many matters besides room density are, of course, covered.

Following are the British instructions to enumerators as given in the

[1] *Les Méthodes de la Statistique de l'Habitation*, pp. 39, 40.

[2] *Ibid.*, pp. 40, 41.

[3] *Ibid.*, pp. 41, 42.

[4] "On comptera comme pièce tout compartiment d'une maison, *destiné à l'habitation*, séparé des autres par des cloisons allant jusqu'au plafond et assez grand pour contenir un lit d'adulte (comprendre la cuisine, l'antichambre, les cabinets de toilette, assez grands pour contenir un lit, les chambres de domestiques, même séparées du logement, mais non les lieux d'aisance; ne comprendre les boutiques, ateliers, écuries, etc., que lorsque une ou plusieurs personnes y passent habituellement la nuit)." Modèle No. 4, Dénombrement de 1926, Bordereau de la Maison.

[5] *Les Méthodes de la Statistique de l'Habitation*, p. 39.

[6] Schedule 2, Dwellings. *Folketelling i Norge*, 1 desember 1920, Skjema 2, Husholdningsliste.

Enumeration Book, Census of England and Wales, 1921, p. VII, paragraph 19:

> *Number of Living Rooms.* He (the enumerator) must also, before delivering the Schedule, enter in the space provided at the foot of the inside portion the number of living-rooms occupied by the persons to be enumerated on the Schedule, and must ask any questions necessary to enable him to ascertain that number. The kitchen should be counted as a room, but the enumerator must *not* count a scullery, landing, lobby, closet or bathroom, or any warehouse, office or shop rooms.

For simplicity and clearness, this could hardly be improved upon. The precise list of examples not to be included in the room count would differ somewhat, however, in every country, according to local building habits.

In the Detroit School Census taken in May, 1928, a family schedule was used, as in England and various other countries. On it one square inch of space is devoted to what it calls *Housing Inquiry*, with *Number of Rooms* and *Number of Occupants* as sub-headings. In the instructions to enumerators, under *Housing Inquiry*, we find the following:

> As directed by the Board of Education there has been included on the 1928 field sheet an inquiry for the purpose of securing data showing the housing conditions at this time. Return must be made and housing data supplied for every dwelling, apartment, flat, or business property in your district even though no children of census age live there. The first question "Number of Rooms," would include the number of rooms used as bedrooms, living rooms, kitchens; and exclude places where business or industry is carried on, or which are used exclusively as laundries, as well as halls, bathrooms, store rooms and closets. The second question, "Number of Occupants," would correspond to the term "census family"; that is, the number of actual residents in the house or dwelling place, whether related to each other or not.

As the inquiry is new in the United States, in order to insure greater accuracy, a loose sheet was also furnished to the enumerators containing the following *Information as to What Constitutes a Room:*

> Borderline cases often present difficulties. The extremes of a series are easy to classify, but not the ones in the middle. A few examples will illustrate. An electric plate in a closet does not make it a kitchen, nor do an electric plate and a sink in a pantry. But at some point a kitchenette is evolved which, though small, is actually a room where the family meals are prepared. Adding a few square feet to a hall, putting in a chair, a rug and a picture and calling it a reception hall do not make it a room. Yet a large reception hall, containing the stairway to the second story, may be the actual family living room. Somewhere in between will be a case hard to decide. Chairs or a cot on a porch in summer do not make it a room, but some porches are glassed in, plastered, provided with steam heat and used all the year round as bedrooms or sitting-rooms. They should be so counted. The so-called dining-nook may raise a question. It is

not a separate room unless it is enclosed on all four sides by partitions which reach the ceiling. When one whole side of the dining-nook is open, or open above a low barrier, it is a part of the room adjoining. A similar distinction sometimes helps in the matter of alcoves. A given space should be classed as a separate room rather than an alcove, if it has partitions reaching to the ceiling on all four sides, even though the partitions may not completely close all sides, provided it has at least one window to afford independent lighting and ventilation. Common sense will usually answer the question in a given case: Is this space an independent room? And is it actually being used for living purposes?

In order to make the try-out as helpful as possible to the Census Bureau or to other municipalities, blanks were distributed to the enumerators asking them to fill in at the close of their work whether they had experienced any special difficulty in getting the number of rooms figure correctly, about how many puzzling cases they had had, and whether they had found the instructions clear and adequate. These returns, when available, should be distinctly informative.

A future all-American instruction might contain a few more excluded items than that of Detroit, as, for instance: "Living rooms of all kinds, bedrooms and kitchens are to be counted; but not halls, porches, bathrooms, closets, pantries, alcoves, storerooms, laundries, or work-shops, mercantile establishments or offices." But the supplementary information sheet could be omitted. For, after all, why should we worry about the borderline cases? If they are hard to decide, it is because they come close to ranking 50-50 in living-room and non-living-room characteristics. No serious error, therefore, will be made by classing them either way. Such faulty exercises of judgment as occur will tend, in the long run, to balance each other. Moreover, and this is of great importance, the type of dwelling in which these doubtful classifications occur is not the type inhabited by unskilled wage earners, and it is in the homes of unskilled wage earners—the lowest economic third of the population—that the dangerous over-crowding is found, about which we are seeking information. In other words, any small inaccuracies in density coefficients which these cases may introduce will affect the groups living less than one person to a room rather than those living two or more to a room.

After the material is gathered, the next point of interest is the method of tabulating. There is a handy Report by the Valuer to the Housing Committee of the London County Council on the facts relating to London housing in the 1921 Census, which appeared in advance of Census publications (1923) and may serve as typical. Passing over the tables dealing with population, number and class of dwellings, new dwellings, demolitions, vacancies, houses occupied by two or more families (without structural change adapting them to such use—a group

analogous to the illegally converted tenements of New York), we come to the sections which deal with tenements and families and with over-crowding, "a tenement being a room or rooms, whether comprising a structurally separate dwelling or not, occupied by a single family" (page 8).

The first of this series of tables gives tenements of various sizes in 1921, by number of rooms and inhabitants. The text points out that the greatest number of persons occupy three-room tenements, the next greatest numbers being those who live in four rooms, two rooms and five rooms, in the order named. Of the dwellings themselves, it is observed that 75.8 per cent fall in the one- to four-room group, which are almost exclusively occupied by working class families, while the five- and six-room dwellings, which constitute 15.1 per cent of the whole, are, with almost equal precision, the homes of the middle, or lower-middle class. No such correspondence could be predicated in the United States, where the recent tendency of the fairly well-to-do to seek small apartments has been marked.

Next come comparisons with 1911 figures to show ten-year trends. Changes in the size of families as well as changes in the size of tenements are considered.

Under the head of Over-crowding come the most socially important tables. "In the Census tables of 1911 the standard of more than two persons per room was used as a measure of over-crowding. In the present Census tables this standard has not been used, and in the Census report it is explained that it had 'been commented upon as though it had been propounded as an absolute standard or a definition of over-crowding.' Probably to avoid this criticism a scale on the basis of 'number of rooms per person' has been adopted." So says the *London County Council Report on the 1921 Census*, which we are citing. It goes on to explain that, because of the familiarity of the British public with the older form, it has turned the tables back to their equivalent in "persons per room" (page 10).

As a sample of how much of human interest can be packed into a few lines, take this summary on page 11.

MORE THAN TWO PERSONS PER ROOM, 1921

	Total, private tenements, 1921	More than two persons per room	
		Number	Proportion of total
			Per cent
Tenements	1,120,897	110,496	9.86
Rooms	4,057,271	244,707	6.03
Persons	4,243,838	683,498	16.11

Next come tables showing in detail, for 1911 and 1921, the number and percentage of persons living more than two to a room in tenements of various sizes and also (Appendix G) the number of tenements and of rooms occupied in this manner. Next are shown the number of persons living at even higher densities—more than three to a room, more than four, etc., till we reach twelve (two cases of thirteen each). A geographical analysis follows for the various metropolitan boroughs. Cases of exceptional over-crowding are tabulated. Decrease in the proportion of children to adults is noted. The one- to four-room tenements are considered separately. The slow, but steady decrease in over-crowding since the 1891 Census is demonstrated. The density in families of various sizes is shown to be greatest in the large families. Thus, less than 4 per cent of families of four are over-crowded, but 18 per cent of families of five, 56 per cent of families of nine, and 80 per cent of families of thirteen suffer in this manner. Comparisons are made between the density conditions in London and in the industrial cities of Leeds, Birmingham, Liverpool, Sheffield and Manchester, none of which is as badly off as the metropolis.

The more we study this wealth of statistical material, all derived from the single additional inquiry, number of rooms per family, the more clearly we perceive as one of the major reasons for our comparative backwardness in adopting comprehensive measures for the solution of our housing problems, our total lack of the statistics of room congestion. To fight an evil effectively, we must know where to find it and how much there is of it.

THE SIGNIFICANCE OF FIXED-BASE AND LINK RELATIVES IN STUDIES OF PRICE STABILITY—A COMMENT ON *THE BEHAVIOR OF PRICES*[1]

By Dorothy C. Bacon, *Smith College*

"There is, of course," says Dr. Mills, "a close relation between the problem of price stability and that central problem of economic theory which is concerned with general economic equilibrium. In studying price stability we are dealing with the monetary aspects of that classical problem."[2]

The study of these aspects of the classical problem might be carried on in two different ways. The first, theoretical, would set up a standard for the conditions of equilibrium and then, by investigating various forces which might play on this equilibrium, would develop the types of price structure which would arise under given conditions. The second, statistical, would describe the types of price structure which have actually developed when various forces have been influencing the equilibrium. The second is the method of *The Behavior of Prices*. Now the first method is of no significance unless it is found to tie up with the results of actual experience. On the other hand, the results of the second method are of little value unless they can be interpreted in some theoretical manner. Dr. Mills has not related his statistical results to theories of the price structure in this volume. Undoubtedly this will form a part of the promised second volume. In the meantime it is interesting and important to consider the theoretical implications of the statistical measures developed.

In the greater part of his investigation Dr. Mills has found that prices in their absolute form could not be utilized for they are not directly comparable due to differences in the units for which they are quoted and other such factors. For this reason he has been compelled to express them in relative form. The types of relatives are fixed-base and link relatives. The reason given for the use of these types is best shown by the following quotation dealing with the use of link relatives as a measure of year to year variability.

> The practical consideration (ease of computation) was reënforced by theoretical considerations. In measuring price variability it seems desirable to take

[1] The writer wishes to express her indebtedness to Professor Warren M. Persons for guidance in her general study of price phenomenon and for criticism of this manuscript. The writer, however, accepts sole responsibility for all opinions expressed in the article.

[2] Frederick Mills, *The Behavior of Prices*, National Bureau of Economic Research, 1927, p. 217.

> account of the actual changes which occur from year to year, for it is these changes which affect business dealings and economic relations. The chief alternate measure, based on deviations from a line of trend, involves the use of a somewhat arbitrary base point for each year's calculations, a base point which becomes quite artificial and meaningless unless the fit of the trend line is satisfactory. Even when the fit is good it is questionable whether a measure of variability of a time series should be based upon deviations from a line of trend. Actual fluctuations, not departures from a hypothetical trend are the significant manifestations of variability. In these computations, accordingly, price changes from year to year are employed, instead of deviations from lines of trend.[1]

This quotation gives no very positive reason for the choice. The argument seems to be that this method is good because another is bad. The other method is called artificial, arbitrary, and difficult to do well. But these statements are not corroborated by any reasons. The fact that the author goes on to show that one method approximates the other (in what later proves to be the typical case) makes one feel that he himself is in some doubt. Later in the book when measures of group relations are presented we are told that it is "the degree or rapidity of change rather than the degree of departure from any hypothetical state of normality which can be measured."[2] Still fixed-base and link relatives are used, though without further discussion of the choice. Change must be a change from something, and in measuring it there must be a zero point. For this reason it seems desirable to consider with just what kind of zero point we are dealing when we use fixed-base and link relatives.

We will begin by discussing the fixed-base case and then take up the case of link relatives as modifications of our discussion. When we choose a fixed base we choose the price of a commodity for a given month, year, or period of years, and call it 100. Thereafter the price of the commodity is expressed as a percentage of that original price. If the base price happens to be unusually high, all subsequent fluctuations will seem small because they are percentages of a large quantity. On the other hand if the base is small, all other fluctuations will appear large. The base price must be kept in mind constantly in interpreting the results.

The base price must in no wise be connected with the concept of normal value of economic theory nor of normal value, expressed in monetary units. It is true that we usually try to pick a base price which will be a "normal" price for the series. But this is different from the normal price of economic theory in two respects. In the first place since the economic world is not in the static state, we cannot

[1] Frederick Mills, *The Behavior of Prices*, National Bureau of Economic Research, 1927, p. 50.
[2] *Ibid.*, p. 74.

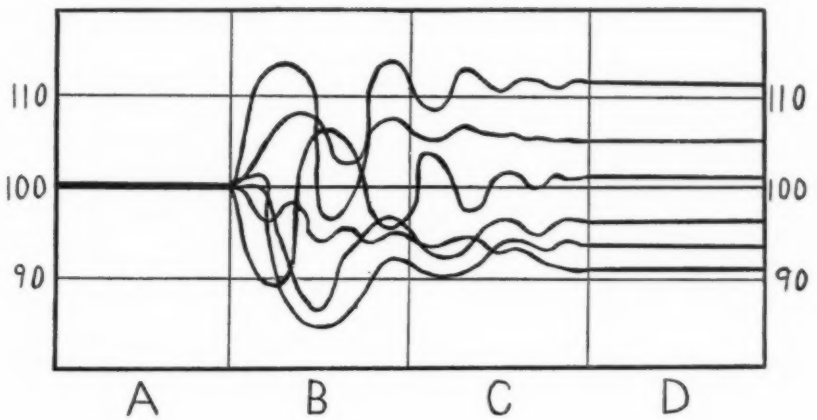
think of the economic equilibrium as fixed. It is constantly changing —a moving equilibrium. For this reason, even though the base price were the normal price of the time, it would not be the normal price of any other time. In the second place it is probable that an actual price is seldom equal to its theoretical normal price. The very shifting of the equilibrium itself may set up conditions which would make the actual price deviate from normal—economic friction, errors of optimism and pessimism and what not—quite apart from other disturbing factors which are always present in a dynamic society. Our fixed-base relative, therefore, is a relative to a price which is not normal and which tends to be farther and farther from normal as the interval of time becomes longer and longer. It is for this reason, undoubtedly, that Dr. Mills changes to a new base every ten years approximately. During each ten year period, however, the difficulty is cumulative. This method also prevents the direct comparison of fluctuations of one period with fluctuations of another period for each represents a percentage of a different base.

The difficulties pointed out in the previous paragraph become yet more serious when we leave the consideration of individual series and pass to that more complicated problem of measures of prices in combination and price stability. It will be remembered that Dr. Mills forms frequency distributions of relatives—cross sections of the price structure as it were—and computes various statistical measures descriptive of their shape—dispersion, skewness, kurtosis—and from these he attempts to draw conclusions as to the stability of the price structure. In the use of fixed-base relatives for such groups of prices, the period chosen may yield an approximately "normal" price for only certain of the commodities to be described. Commodities are affected in different degrees and for different lengths of time by various general price disturbances, besides the peculiar factors of each individual market. So for any particular period we would expect to find some prices high above, some prices near, some prices far below their normal price. But what does our fixed base do to these prices? *It makes them appear all exactly alike for the base period.* The fixed base removes all clues to the real situation. Only by a detailed study of the actual prices themselves in their absolute form can the true price structure be discovered. And not only are the relative differences obscured, but the size of all later fluctuation will be determined by the size of these bases, differing from commodity to commodity.

Let us examine this situation a little more closely. We may well begin with a simple and necessarily artificial case. Suppose the economic world were in a static state, such as Marshall so admirably

describes, of which the chief characteristic, as far as we are concerned, is that all prices are and long have been at their normal price. Now let one sudden change take place—perhaps the normal rainfall changes. What will happen to the price structure as measured by fixed-base relatives? We begin with all prices at 100. The change comes and costs of production change in agriculture. The disturbance spreads to other industries. The effects of the change are different in different industries due to varying degrees to which the costs are increased or decreased and to the different elasticities of demand which are met in the market. Errors of optimism and pessimism, etc., enter in at the start, but given a sufficiently long time with no new disturbances the world will again settle down to another static state, but all economic relations are changed. The price structure might be as in Chart I. A here represents the original static state, B, and C periods

CHART I

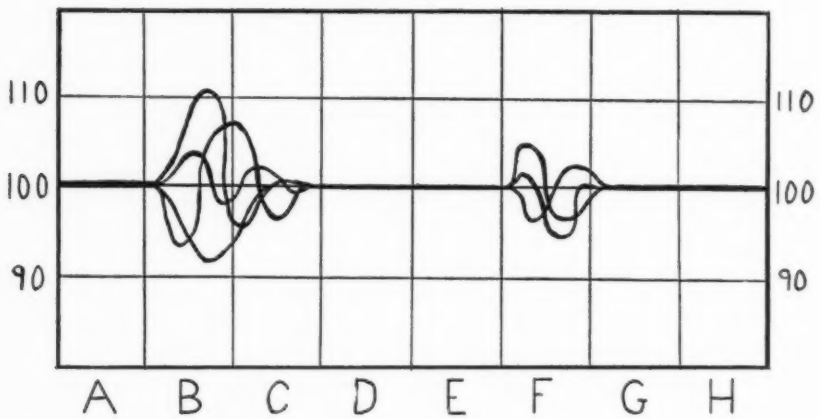


of disturbance, D another static state. As measured in terms of fixed-base relatives D and A are widely different. All measures of group relations would be different. Yet as far as the economic significance is concerned A and D are exactly alike. Both are static states. True, factors of production are combined in different degrees, prices are different, but no element of instability is present in the price structure. The fixed-base relatives show the change that has taken place as compared to the old normal price. But from the fact that some relatives are high and some are low and dispersion is great, one can infer nothing as to the stability of the new price structure. If D had been chosen as a base, the prices in A would have appeared dispersed.

Now let us take another case. Suppose the change had been tem-

porary and recurring at irregular intervals. The results would be the same except that between each disturbance, granted a sufficient period of time, we would return to the same equilibrium state. The price structure might appear as Chart II.

CHART II



In this case every equilibrium state is expressed by 100 and dispersion of relatives in the disturbed periods would mean instability. But suppose we had chosen B as our base period. Then just the opposite would be true. The more alike prices appear to be the more disturbed they really are. The more dispersion there appears to be, the more normal the price structure is and the more stable the price level, if no new cause of disturbance arises.

Both of these cases are artificial and not typical of our actual world. But they represent the sort of thing that is continually taking place. Instead of normal price shifting once and for all because of a change in one or a group of commodities, it is continually changing because of changes in many costs of production and these changes are constantly recurring. Superposed upon this is the temporary change of Chart II. But again the change may emanate from many commodities. These changes will follow in rapid succession with no return to an equilibrium state between.

Moreover, changes in the quantity of the circulating medium—currency and credit—will tend to move the general price equilibrium and also to generate short time disturbances. Just exactly in what way these factors enter in need not concern us here. It is simply one more contributing factor, differing in that it possibly affects a larger number of commodities at the same time and causes the center of the price system to drift in one direction or another.

But what, in this actual state, can we expect of our fixed-base relatives? They are influenced by two sets of factors: factors changing the normal price of each series, and factors tending to deflect them from that normal price. The base may well represent a much greater deflection from normal in one case than another. All changes must be interpreted in this light. Would increasing dispersion of relatives probably mean greater or less stability? It depends on the base. If the base chosen is like period B of Chart II it would mean greater stability. It might be argued that no one would choose a period such as B as a base for a price index. But are actual choices any better? Dr. Mills uses such years as 1891 and 1902. Turning to the annals of the period we find such statements as these:

> The first seven months of 1891 in America was a period of liquidation after the crisis of 1890. But mid-summer brought a dramatic change in the business situation. As in 1879 the country was suddenly lifted from depression to prosperity by the concurrence of bad harvests in Europe and abundant harvests in America. The first beneficiaries of this stroke of fortune, the farmers and railways, bought goods with freedom stimulating trade, manufacturing, mining, etc., in almost all lines. . . .
>
> The contrast between abounding prosperity in general business and troubled financial markets continued through 1902. Volume of trade was declared greater than ever before—Railways were offered more freight than they could carry—The Steel corporation and Pennsylvania Railroad made voluntary increases of 10 per cent in wages—Farmers had brilliant harvests—Even the high price of meat which followed from the deficient stock of corn and the great Anthracite strike of May to October did not stem the rising tide. But financial reports betrayed signs of increasing strain. . . .[1]

Such are two of the base years. The description hardly suggests equilibrium conditions. Averaging long periods would not help matters. It would simply make the difficulties less obvious. No fixed base can catch the price structure in equilibrium.

This discussion does not mean that fixed-base relatives are of no value. They present a picture of percentage changes from a given date, and this may be our interest in some studies. But when we are interested in such matters as price stability and price dispersion and displacement, we must remember that the significance depends on the base. Meat drops in price from 100 to 65. Does this mean that there is a cause of instability in the price level, if other prices average 115? Not if 1902 was the base year with meat at a premium due to a scarcity of corn. But if wheat did the same thing, it might be an entirely different story. No measure of dispersion developed from fixed-base relatives can measure forces working towards instability, nor can

[1] W. C. Mitchell, *Business Cycles*, 1913, pp. 51 ff.

other measures of the form of the distribution of such relatives. *The trouble is inherent in the relatives themselves.*

Link relatives differ from fixed-base relatives in that the base is changed every year or every month. If there were no other forces at work than those which change equilibrium price, the base could never be very far removed from the normal price. If the normal prices change at a constant rate of increase or decrease,[1] the distribution of relatives would be the same always. If the shift takes place in any other manner the distribution of relatives might assume changing forms without any significance as to price stability. When other causes of change are added the situation becomes yet more complicated. Each link relative depends not only on what the price in the given month or year is but also on that for the previous month or year. If the price is near the normal price for several consecutive months the relatives will be near 100, if very high for several months the same thing will happen. A 100 value may mean one thing for one commodity, another for another. A relative of 110 may mean an increase in stability if the price of the commodity had been previously low, away from stability if it had been previously high. These difficulties will be compounded with those arising from the shifting of the normal price. Just as in the fixed-base case, the distribution of relatives will not be significant of price relationships. We do not wonder that we find Dr. Mills saying, "There is no evidence here of a consistent relationship between changes in the price level and changes in the degree of dispersion."[2]

After condemning these two types of relatives for the purpose of studies of price stability, we do not like to leave the question without some constructive suggestion. It would seem from the preceding discussion that we must make some attempt to find the moving normal price for each series for each date and express each price as a relative to that normal price. The relatives would then be freed from the peculiar conditions of any particular time. Such a moving normal price would seem to be furnished by the values of the trend line for each series.[3] Dr. Mills has characterized such a trend line as arbitrary. It does, indeed, involve the element of judgment. Yet most statistical work of value depends on judgment to some extent. The absolute prices we begin with are not chosen at random, neither are the bases. In fact fixed-base and link relatives measure from as arbitrary points as do percentage-of-trend relatives. The only difference is that

[1] Mills, *op. cit.*, p. 50.

[2] *Ibid.*, p. 283.

[3] See H. L. Moore, "A Theory of Economic Oscillations," *Quarterly Journal of Economics*, November, 1926.

a somewhat more complicated formula is used in one case than in the other.

And Dr. Mills' book, in spite of the passage quoted above, is full of suggestions towards the use of percentages of trend. Time and again we fully expected to see the percentage-of-trend relatives introduced on the next page. For instance we find such a statement as this:

> . . . It does not appear justifiable to assume that prices in any one year, or average prices over any period of years, stand in a "normal" relation to each other. There was no formal pre-war relation between prices in their absolute form. The only normal factor bearing on pre-war price relations (if we mean by normal a situation sustained in a fairly regular fashion over a number of years) was the existence of fairly constant rates of change in individual commodity prices, rates of change which differed materially from commodity to commodity and from group to group. What was constant in the pre-war price situation was not a set of fixed price differences but relations which changed at fairly regular rates year by year. . . . To assume that actual prices in a given year stand in a normal relation to each other is to crystalize a cross-section of a constantly changing situation, a procedure which seems equally faulty whether a single year or a number of years be used in computing the basic averages.
>
> When an average annual rate of change in the price of a given commodity has been sustained over a considerable period of years it is reasonable to assume that this constant annual increment (or decrement) in price comes to be expected by those handling this commodity. The effects of accidental and cyclical forces are, of course, superimposed upon the tendencies due to trend, so that there may be no conscious expectation of a certain annual change. Nevertheless, the fact of a long sustained tendency to increase or decrease will have affected the attitudes of manufacturers and dealers and will be reflected in their business dealings and expectations.
>
> . . . There thus develops a sense, not of what constitutes a normal relation between prices, but of the alterations in relations to be expected in the future. When such expectations are not realized—there arises the feeling that normal price relations have been shattered. Though the concept of normal relations be illusory, the feeling of disturbance and unsettlement arising from the failure of past tendencies to continue may be a very real one.[1]

The expected prices of this discussion would seem to offer an approximation to the normal price we seek. They are the prices on which the business man depends in planning his output. With relatives expressed as percentages of these prices we should have an opportunity to judge of the elements of instability in the price structure. We would venture to suggest that a study of such relatives would be more significant for problems of price stability.

[1] Mills, *op. cit.*, pp. 74 ff.

THIRTEEN-MONTHS-RATIO-FIRST-DIFFERENCE METHOD OF MEASURING SEASONAL VARIATION[1]

By A. O. Bauman, *Research Division, Armour and Company*

There are four well-known or commonly used methods of computing correction for seasonal variation in time series data, namely, (1) arithmetic averages of monthly items, (2) twelve months moving average, (3) ratio to trend, and (4) link relative. Many students of statistics have appreciated some disadvantages of all of them. The first two have been practically superseded by the latter two. This discussion, therefore, will be limited to the ratio-to-trend and link-relative methods of computing seasonal variation together with a comparison with a new method, which may be called the thirteen-months-ratio-first-difference method.

The discussion will be under the following headings: (1) Disadvantages of (a) Ratio-to-trend method, (b) Link-relative method; (2) Thirteen-months-ratio-first-difference method; and (3) Advantages of the new method.

(1) DISADVANTAGES OF (A) RATIO-TO-TREND METHOD

The subjective element involved in fitting the trend line in the ratio-to-trend method has been offered as an objection, as well as the lack of emphasis upon month-to-month changes. It seems that a more important objection is the unsuitability of the trend ordinates as a base to which actuals are to be related. When the fluctuations are at a considerable level above their respective trend points, although the whole period is seasonally highly typical, the resulting ratios will, nevertheless, take a high position in the frequency table set up for the selection of an average. Similarly, when the fluctuations are at a considerable level below their respective trend points, although again the whole period is seasonally highly typical, the resulting ratios will take a low position in the frequency table. Whenever the fluctuations are close to the trend line, although maybe not seasonally characteristic, the ratio may, nevertheless, occupy a central position in the frequency table and therefore be used. In other words, characteristic seasonal changes remote from an estimated trend line are penalized as com-

[1] The idea of first differences is not new in computing seasonal variation, and the writer acknowledges the article by F. L. Carmichael in the September, 1927, issue of this Journal. Also see articles by Professor Persons on the link-relative method, Helen D. Falkner on the ratio-to-trend method, and other articles.

pared with seasonal changes perhaps less characteristic but nearer the trend line. The ratio-to-trend method is, therefore, not precise or satisfactory as a measure of seasonal variation.

The same problem of selecting a base is involved in the link-relative method. In the new method, to be explained, and in fact in every calculation using ratios or relatives the problem is to find a base against which the fewest objections may be advanced.

(B) LINK-RELATIVE METHOD

The link-relative method was invented chiefly to measure relative month-to-month changes, at the same time permitting correction for trend. The base used by this method is the figure of the preceding month. Assuming the preceding month may be a criterion for measuring the value of the following month a shifting base as represented by the link-relative denominator distorts the ratio out of proportion to the month-to-month changes. For example, while a given month may be seasonally representative but the preceding month accidentally high or low, the effect is that the resulting relative is thrown to an extreme position in the frequency table used as a basis for determining the median or whatever average is used. The net result is increased dispersion allowing a wider latitude as to the probable correct value of the median. Inasmuch as the precision of the central item is determined by the dispersion of the series, a method giving increased dispersion, in effect, is equivalent to reducing the number of items. Using the same number of items, diminished dispersion makes for increased precision, and increased dispersion lessens precision.

Furthermore, while the invention of the link-relative method facilitates correction for the trend and emphasizes month-to-month changes, the procedure introduces bias as will be apparent later in the analysis. Since the definition for seasonal variation is that it represents monthly departure from the average of the year, presuming, therefore, a fixed base, the question arises whether it is advisable to fix a base in an earlier step of a method or toward the end as is done in the link-relative method.

(2) THIRTEEN-MONTHS-RATIO-FIRST-DIFFERENCE METHOD

The method proposed in this discussion is a result of considerable experience with agricultural data in which seasonal influences are very pronounced. Out of this experience a new method of computing seasonal variation has been evolved and may be described as the "thirteen-months-ratio-first-difference method." This method proposes to measure month-to-month changes without the bias inherent in the

ratio-to-trend and the link-relative methods, at the same time permitting the correction of trend influences. Briefly the steps are:

(1) Compute the monthly mean, median or other average for each year. The year may be the calendar year or the crop year, whichever is more appropriate as a base.

(2) Relate in percentages the actuals of each year to the monthly average or a selected average or median of the year, whatever is more representative, as a base.

(3) Include in the annual series the ratio of the following January, making thirteen months altogether for each yearly series. The thirteenth month is not to be included in monthly averages.

(4) Arrange these ratios in rows, making allowance for thirteen columns, namely, the ratios from January to January, one row for each year, and one column for each month. (See Table II, A.)

(5) Arrange the ratios in the January array in the first column, the differences between the February and January ratios in the second column. Assign positive signs to these differences whenever the February value is greater than the January value and negative signs whenever the February value is less than the January value. In like manner compute the differences between March and February for the third column. This operation is continued until the thirteenth column is obtained showing the differences between January, the thirteenth month, and December, the twelfth month. We now have a second group of figures (see Table II,B) differing from the first group and in the form of month-to-month differences, with the exception of the first month of the year, which, of course, are the ratios of the January values to the monthly averages of their respective years.

(6) Arrange each monthly column of Table II, B in order of size in a new table. (See Table II, C.)

(7) Select a median or other suitable average for each month. (See row A, Table II, C.) Use of a frequency table is helpful in selecting averages.

(8) Cumulate the first differences to the initial January average. (See row B, Table II, C.) The differences between the first and thirteenth month, namely the two Januarys, should be a reasonable measurement of the trend influences with little bias.

(9) Take the difference between the first and thirteenth month, namely the two Januarys, and divide by 12.

(10) In the next row, designated as adjustment for trend, cumulate the average "error" obtained in (9), assigning zero to the initial January value. (See row C, Table II, C.)

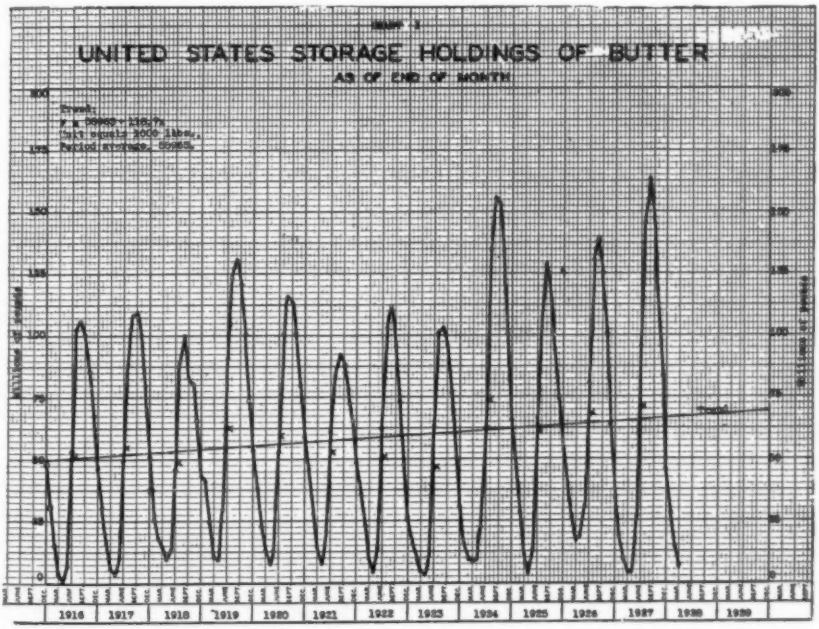
(11) Subtract or add these cumulated "errors" from the respective

items in row B accordingly as the difference between the first and thirteenth month is positive or negative, the resulting values being the selected averages or medians adjusted for trend differences. This makes the two January values equal.

(12) Adjust the first twelve values obtained under (11) to an average of 100 by dividing each month by the average of twelve months. The thirteenth month, of course, should be disregarded in computing the average as it was used to adjust for trend.

By way of illustration, the above method has been applied to United States storage holdings of butter. (See Table I and Chart I.) This

CHART I



series was selected chiefly for the reason that it has a very marked seasonal variation, and an arithmetic trend upward amounting annually to about 2.4 per cent of the average of the entire period. Other series could have been used having more pronounced accidental movements, but in this particular problem we were interested in correction for trend. Where series are used with marked accidental movements the difference between the two January values according to the link-relative method was much greater than accounted for by the trend and it was easily established that these large differences were due to bias resulting

TABLE I

UNITED STATES HOLDINGS OF STORAGE BUTTER AS OF END OF THE MONTH

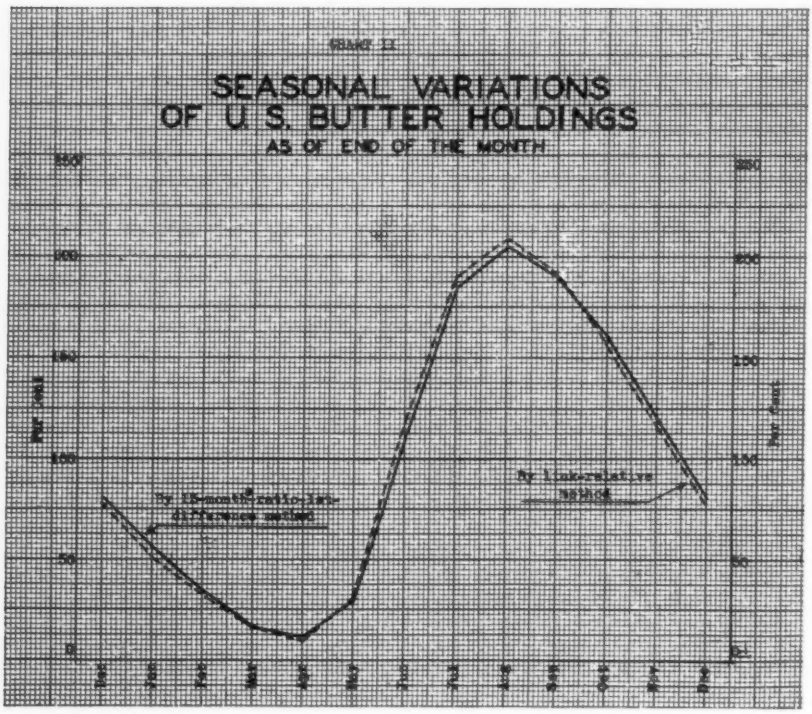
(To nearest 1000 lbs.)

Source: U. S. Yearbook of Agriculture

	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928
Dec. '15	48,977												
Jan.	31,139	30,474	26,616	36,777	38,359	41,486	35,047	16,122	15,246	45,748	39,581	17,967	28,296
Feb.	15,033	16,952	18,808	24,191	22,568	27,103	22,562	8,910	9,647	29,789	26,313	8,066	14,410
Mar.	3,346	6,805	14,629	11,909	12,555	14,732	9,113	4,824	7,842	10,875	17,392	3,033	5,847
Apr.	1,062	3,607	9,536	9,659	7,554	9,712	3,830	3,248	6,912	3,739	17,587	3,432	
May	7,017	9,955	12,698	29,435	12,872	21,682	13,202	10,112	22,348	13,036	30,561	25,340	
June	53,963	49,982	49,140	90,158	52,526	61,991	67,410	62,760	74,184	63,687	66,897	90,116	
July	102,537	88,992	88,305	123,546	101,455	82,636	103,151	101,774	134,118	109,075	131,152	145,146	
Aug.	105,836	108,179	99,334	131,366	115,558	92,292	112,039	102,731	156,440	128,403	136,151	165,037	
Sept.	100,522	109,154	87,863	121,816	113,365	90,116	96,680	96,117	155,494	114,172	125,342	147,412	
Oct.	85,260	100,115	80,874	100,474	101,778	77,983	73,857	76,472	135,018	94,916	100,871	118,768	
Nov.	67,892	79,928	65,111	73,684	79,750	65,189	47,773	51,508	100,832	74,784	84,361	83,340	
Dec.	46,134	50,726	43,910	53,737	56,662	40,412	26,819	30,299	65,694	52,765	54,355	46,309	
Total	619,061	654,867	596,846	808,744	717,042	651,478	611,503	564,865	865,976	739,979	812,383	851,866	
Avg.	51,588	54,472	49,737	67,229	59,754	52,623	50,959	47,074	73,665	61,665	67,694	70,991	

from the use of link ratios. In a certain economic series differences as high as 90 per cent have been observed between the two Januarys by the link-relative method whereas actual growth amounted to only about 3½ per cent, short-time influences having been insignificant. For a calculation by the link-relative method see Table IV.

CHART II



(3) ADVANTAGES OF THE NEW METHOD

Table III shows results obtained by the new method and the link-relative method. (See Chart II.)

It will be noticed that the difference between the two methods varies as much as 5.6, for June. This amounts to 25 per cent of the standard deviation from the computed normal of the data. In comparing the differences between the January values of the chain relatives by the

TABLE II

SEASONAL INDICES (13 MONTHS-RATIO-FIRST-DIFFERENCE METHOD)

Subject: U.S. Holdings of Storage Butter

A. Ratios to Average of the Year

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.
1916	60.4	29.1	6.5	2.1	13.6	104.4	196.6	205.1	194.9	185.3	130.4	89.4	59.1
1917	55.8	31.1	12.5	6.6	18.2	91.6	165.1	198.1	200.0	183.5	146.5	93.0	48.8
1918	53.5	37.8	29.8	19.2	25.5	98.8	177.5	199.8	176.7	162.6	130.9	88.3	73.7
1919	54.7	36.0	17.7	14.4	43.8	134.1	183.8	195.3	181.2	149.5	109.6	79.9	57.1
1920	64.2	37.8	21.0	12.6	21.5	87.9	169.8	193.4	189.6	170.3	133.5	96.2	69.4
1921	78.8	51.5	28.0	14.7	41.2	117.8	157.4	175.3	171.3	148.2	123.6	92.0	66.6
1922	68.6	44.3	17.9	7.5	28.9	152.5	208.4	219.9	189.7	144.9	93.8	52.6	31.6
1923	34.5	18.9	10.3	6.9	21.5	133.3	216.2	218.1	204.2	162.5	109.4	64.4	32.4
1924	20.7	13.4	10.6	12.1	30.3	100.7	182.1	212.3	208.4	183.3	136.4	89.2	62.1
1925	74.2	46.7	17.6	6.1	21.1	108.5	176.9	208.7	186.8	153.9	121.8	85.6	65.9
1926	58.2	36.9	25.7	25.9	45.2	139.2	193.7	204.0	186.1	149.0	96.1	50.8	38.5
1927	25.5	11.4	4.3	4.8	33.7	186.9	204.5	229.4	207.7	167.5	119.3	65.2	39.9

B. First Difference

1916	60.4	-31.3	-22.6	-4.4	11.5	90.8	94.4	8.5	-10.2	-29.6	-54.9	-41.0	-30.3
1917	55.8	-24.7	-18.6	-5.9	11.6	73.4	71.5	33.0	1.9	-16.5	-37.0	-53.5	-44.2
1918	53.5	-15.7	- 8.0	-10.6	6.3	73.3	78.7	22.3	23.1	-14.1	-31.7	-42.6	-14.6
1919	54.7	-18.7	-18.3	- 3.3	29.4	90.3	49.7	11.5	-14.1	-31.7	-39.9	-29.7	-22.8
1920	64.2	-26.4	-16.8	-8.4	8.9	66.4	81.9	23.6	3.6	-19.3	-36.8	-37.3	-26.8
1921	78.8	-27.3	-23.5	-13.3	26.5	76.6	39.6	17.9	- 4.0	-23.1	-24.4	-31.6	-25.4
1922	68.6	-24.3	-26.4	-10.4	19.4	108.4	70.1	17.5	-30.2	-44.8	-51.1	-41.2	-21.0
1923	43.5	-15.4	- 8.6	- 3.4	14.6	111.8	82.9	1.9	-13.9	-14.7	-55.1	-45.0	-32.0
1924	20.7	- 7.3	-2.8	1.5	18.2	70.4	81.4	30.2	- 3.9	-25.1	-46.4	-47.2	-27.1
1925	74.2	-27.5	-29.1	-11.5	15.0	88.2	73.6	31.3	-23.0	-31.3	-32.7	-35.6	-21.7
1926	58.2	-19.3	-13.2	.2	19.3	93.0	65.5	10.5	-18.9	-36.1	-53.9	-44.3	-24.3
1927	25.5	-13.9	- 7.1	.5	30.9	91.2	77.6	25.1	-21.9	-40.4	-50.0	-54.1	-25.3

C. Above arranged in order of magnitude

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.
1	20.7	-31.3	-29.1	-13.3	6.3	66.4	39.6	1.9	-30.2	-44.8	-53.9	-53.5	-44.2
2	25.5	-27.5	-26.4	-11.5	8.9	70.4	49.7	8.5	-23.1	-41.7	-55.1	-52.1	-32.0
3	34.5	-27.3	-23.5	-10.6	11.5	73.3	65.5	10.3	-23.0	-40.4	-51.1	-47.7	-30.3
4	53.5	-26.4	-22.6	-10.4	11.6	73.4	70.1	11.5	-21.9	-36.1	-50.0	-45.0	-29.8
5	54.7	-24.7	-18.6	- 8.4	14.6	76.6	71.5	17.5	-18.9	-31.7	-46.4	-44.3	-27.1
6	55.8	-24.3	-18.3	- 5.9	18.0	82.2	73.6	17.9	-14.1	-31.3	-39.9	-42.6	-25.4
7	58.2	-19.3	-16.8	- 4.4	18.2	83.0	77.6	22.3	-13.9	-29.6	-37.0	-41.2	-25.3
8	60.4	-18.7	-13.2	- 3.4	18.4	90.3	78.9	23.6	-10.2	-25.1	-36.8	-41.0	-24.3
9	64.2	-15.9	- 8.6	- 3.3	19.3	90.8	81.4	25.1	- 4.0	-23.1	-34.9	-36.6	-22.8
10	68.6	-15.4	- 8.0	.2	26.5	91.2	81.9	30.2	- 3.9	-19.3	-32.7	-35.3	-21.7
11	74.2	-13.9	- 7.1	.5	29.4	108.4	82.9	31.3	- 3.6	-16.5	-31.7	-31.8	-21.6
12	78.8	- 7.3	- 2.8	1.5	30.9	111.8	94.4	33.0	1.9	-14.1	-24.4	-29.7	-14.6
A Aver.(Median) 1st Difference	57.0	-21.9	-17.8	- 5.2	16.6	82.6	75.6	20.1	-14.0	-30.4	-38.5	-41.9	-25.4
B Cumulative 1st Differences	57.0	35.1	17.5	12.3	28.9	111.5	187.1	207.2	193.2	162.8	124.3	82.4	57.0
C Trend	0	0	0	0	0	0	0	0	0	0	0	0	0
D B - C	57.0	35.1	17.5	12.3	28.9	111.5	187.1	207.2	193.2	162.8	124.3	82.4	57.0
E Seasonal Variation	36.1	34.8	17.2	18.1	28.4	109.1	184.2	204.0	190.2	160.2	122.3	81.1	

link-relative method, with the January differences by the thirteen-months-ratio-first-difference method, it will be noticed that the former shows a difference of 20.1 units and the latter no difference. The

TABLE III

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
New method.....	56.1	34.5	17.2	12.1	28.4	109.7	184.2	204.0	190.2	160.2	122.3	81.1
Link-relative method.....	51.9	32.6	16.6	10.2	29.4	115.3	189.3	208.4	191.3	158.1	119.1	77.8
Difference.....	4.2	1.9	.6	1.9	1.0	5.6	5.1	4.4	1.1	2.1	3.2	3.3

amount of trend suggested by the link-relative method, namely 20.1 per cent, is considerably greater than the long-time trend as measured by the method of least squares, namely 2.4 per cent. (See Chart I.) [1] The difference between the two Januarys by the thirteen-months-

TABLE IV

SEASONAL INDICES (LINK REL. METHOD)

Subject: U. S. Holdings of Storage Butter

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1916	63.6	48.3	22.3	32.3	648.5	767.6	190.4	105.2	95.0	84.8	76.9	68.6
1917	66.1	55.6	40.1	53.0	275.9	502.2	178.0	121.6	100.9	91.7	79.8	63.5
1918	52.5	70.7	77.8	65.2	133.2	387.0	179.7	112.5	88.5	92.0	80.5	67.4
1919	83.8	65.8	49.2	81.1	304.7	306.3	137.0	106.3	92.7	82.5	73.3	73.0
1920	71.4	56.8	56.6	60.2	170.4	408.1	195.2	113.9	98.1	89.8	78.3	73.6
1921	70.7	65.3	54.4	52.3	361.1	285.9	133.6	111.4	97.6	86.5	83.5	74.4
1922	72.4	64.4	40.4	42.0	344.7	510.6	153.0	108.6	86.3	78.4	64.7	56.1
1923	60.1	55.3	54.1	67.3	311.3	620.7	162.1	100.9	93.6	79.6	67.4	58.8
1924	50.3	64.6	79.6	113.7	250.7	331.9	180.8	116.6	90.1	88.0	74.7	65.2
1925	69.6	62.9	37.8	34.4	348.7	488.5	171.3	117.7	88.9	83.1	78.8	70.6
1926	74.6	66.8	66.1	100.8	174.4	284.3	150.9	105.3	90.7	90.5	83.8	53.6
1927	52.3	45.0	37.5	113.1	736.3	355.6	161.1	112.3	90.4	80.6	70.1	55.6

Above Arranged in Order of Magnitude

1	50.3	45.0	22.3	32.3	133.2	284.3	133.6	100.9	86.3	76.4	63.8	53.4
2	52.3	48.3	37.5	34.4	170.4	285.9	137.0	103.2	88.5	79.6	64.7	55.6
3	52.5	55.3	37.8	42.0	174.4	306.3	150.9	105.3	88.9	80.5	67.4	56.1
4	60.1	55.6	40.1	52.3	250.7	331.9	153.0	106.3	90.4	80.6	70.1	58.8
5	63.6	56.8	40.4	53.0	275.9	355.6	161.1	108.6	90.7	82.5	73.3	63.5
6	66.1	62.9	49.2	60.2	281.1	387.0	162.1	111.4	92.7	83.1	74.7	65.2
7	69.6	64.4	54.1	65.2	304.7	408.1	171.3	112.3	93.6	84.8	78.3	67.4
8	70.7	64.6	54.4	67.3	311.3	488.5	178.0	112.5	95.0	86.5	78.8	68.6
9	71.4	65.3	55.6	81.1	344.7	502.2	179.7	113.9	97.6	88.0	78.9	70.6
10	72.4	65.8	66.1	100.8	348.7	510.6	180.8	116.6	98.1	89.8	79.6	73.0
11	74.6	66.8	77.8	113.1	640.5	620.7	190.4	117.7	98.1	91.7	80.5	63.6
12	83.8	70.7	69.6	113.7	736.3	767.6	193.2	121.6	100.9	92.0	83.5	74.3

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	
Average (Median) Relative	67.8	63.7	51.7	62.7	292.9	397.4	166.7	111.8	93.2	83.9	76.5	66.3	
Chain Relative	100.0	63.7	32.9	20.6	60.3	239.8	399.7	446.9	416.5	394.4	287.3	177.2	120.1
Trend	100.0	101.5	103.1	104.7	106.3	107.9	109.6	111.3	113.0	114.7	116.5	118.3	120.1
Adjusted Chain Relat.	100.0	62.8	31.9	19.7	56.7	222.2	364.7	401.5	368.6	304.6	229.4	149.8	100.0
Seasonal Variation	51.9	32.6	16.6	10.2	29.4	115.3	189.3	208.4	191.3	158.1	119.1	77.8	

[1] $y = 58963 - 118.7x$; unit, 1000 lbs.; x, the month.

ratio-first-difference method amounts to zero, which is below the calculated long-time trend of 2.4 per cent. However, the error is considerably less by this method than by that of the link-relative. Reference to Chart I will show no upward trend for the period 1916–23 inclusive, and the measured trend is entirely due to the higher level of the last four years as compared with the preceding eight, and since medians were used instead of arithmetic averages of the ratios to average of the year, it can easily be seen why there was no correction for growth. This error of 2.4 per cent by the new method, therefore, is more apparent than real and in any case is little due to bias or distortion.

Emphasis is placed upon the fact that the differences between the ratios by the proposed method are arithmetic rather than relative, which accounts for the absence of bias.

While in the link-relative method the base for any month is represented by the preceding month, the thirteen-months-ratio-first-difference method takes for its base the average of the year. The average of the year as a fixed base represents a sort of center of gravity about which the monthly actual values vary. Theoretically and practically this appears to be a better criterion for comparisons than the shifting base of the link-relative method. Again, when the median values are obtained in the concluding steps of the link-relative method January is taken as a base to which all the link-relatives are chained; but, January is representative of only one-twelfth of the total number of items. On the other hand, the median of the January ratios by the proposed method is representative of the entire series for the reason that the base of each January is the average of the year. Moreover, by the link-relative method, if any particular column of monthly items is erratic, its influence will be imparted to the seasonal indexes in the final result through the step of chaining the link ratios. This influence is obviously undesirable as reflected in the differences between the two Januarys. On the other hand, this undesirable feature is less marked in the thirteen-months-ratio-first-difference method.

There is another objection to the link-relative method whenever there is a shift in the seasonal variation. Experience in attempting correction for shifts in seasonal movements for certain agricultural data has shown the link-relative method wholly inadequate, not alone giving inaccurate results, but, in fact, very contrary results as substantiated by the known facts of the industry. The inaccurate results were due to pronounced influence of bias associated with very large ratios obtained by the link-relative method when it came to the procedure of fitting trend lines correcting for seasonal shift. When the thirteen-

months-ratio-first-difference method was used these objections were removed.

There are problems where the preceding month is not a criterion for the given month and it is not necessary to consider the preceding month in calculating a seasonal index. In such cases the calculation of first differences may be omitted from the procedure. In other words, step (5) enumerated earlier in this discussion, may be omitted. This step is justifiably omitted in certain agricultural series.

The new method has practical advantages. In checking the data, inasmuch as ratios to the average of the year are computed, if the ratios of all the months for each year total 1200, the absence of errors is verified. The link-relatives can be checked only by repeating the whole operation. First differences in the new method can ordinarily be calculated by mental operations. While correction for trend in the concluding steps by the link-relative method requires the use of logarithms, adjustment for trend in the thirteen-months-ratio-first-difference method is made by arithmetic accumulations. This is justified by the fact that the method is on an arithmetic basis, the base for any year in the series being fixed.

To summarize, the advantages of the thirteen-months-ratio-first-difference method over the link-relative method are:

1. Theoretical advantages:
   (a) The influence of distortion or bias inherent in the link-relative method is reduced.
   (b) Base values, namely, averages of the year, by the new method are more representative of the data than the shifting denominators of the link-relative method.
   (c) The method is more suitable to correcting for shifts in seasonal movements by reason of the absence of bias.
   (d) If a previous month is not recognized as a criterion for the following month the first difference step in the procedure of the new method may be omitted, which reduces the formula to one which may be described as a thirteen-months-ratio method.
2. Practical advantages:
   (a) The method is simple by reason of speed and ease of checking results. Also, no logarithms are needed to correct for long-time trend.

THE USE OF MACHINE FACTORING IN MULTIPLE CORRELATION

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The necessary labor in a multiple correlation study is so long and tedious that many research workers, even though they fully understand and appreciate this method of analysis, are kept from using it. Calculating machines, either key driven or crank driven, when used as outlined in the Iowa State College bulletin, *Correlation and Machine Calculation*, by H. A. Wallace and George W. Snedecor, make it possible to calculate the usual statistical constants with speed and accuracy but they do not reduce the number of operations. Thus, in a problem having 492 observations and three variables there are 2,952 multiplications necessary. This is a fair sized job under any conditions but especially so if the numbers consist of several digits. This last objection is partially removed by coding or grouping.

Under ordinary circumstances this process of coding is legitimate and, if properly used, will not make any significant difference in any of the results. Of course coding, like most mathematical processes, could be done so crudely that discrepancies would result. Furthermore, there are many research workers who have labored so hard to carry their observations out to several decimal places that they distrust any method which changes either the form or absolute value of a single observation. The process of machine factoring has been developed to eliminate the chance of discrepancies due to improper coding and to preserve the arithmetic accuracy of the original observations.

Before explaining the process of machine factoring and its use in multiple correlation the problem to which it is applied will be presented. The necessary formulas given by Wallace and Snedecor are of the following type:

Mean: $$M_X = \frac{\Sigma X}{n}$$

Standard deviation:

$$\sigma_X = \sqrt{\frac{\Sigma X^2 - (\Sigma X) M_X}{n}}$$

Simple correlation coefficient:

$$r_{XY} = \frac{\Sigma XY - (\Sigma X) M_Y}{\sqrt{\Sigma X^2 - (\Sigma X) M_X}\ \sqrt{\Sigma Y^2 - (\Sigma Y) M_Y}}$$

As an illustration a set of data will be used consisting of 492 observations on the three variables I, H, and Q.[1] Thus, in order to apply the above formulas, it is necessary to find the sum of the squares of the 492 values of I, ΣI^2, the sum of the H squares, ΣH^2, the sum of the Q squares, ΣQ^2, and the sums of each of the following cross-products; ΣIH, ΣIQ, and ΣHQ. As mentioned in the first paragraph, 2,952 multiplications are necessary to secure the sums of the squares and of the cross-products. By machine factoring the number of multiplications can be reduced from 2,952 to 86 and the multiplier will never be larger than 9.

In order to generalize, any individual value of I is represented as $(100a+10b+c)$, any individual value of H as $(10r+s)$, and any individual value of Q as $(10x+y)$ in which a, b, c, r, s, x, and y are digits 0 to 9 inclusive. Any definite value of c such as 0 will be represented as c_0 or 9 as c_9 and the definite values of the other digits will be represented in the same way. Using these symbols, ΣIH becomes $\Sigma(100a+10b+c)$ $(10r+s)$ which, for reasons that will appear later, will be written $\Sigma(10r+s)$ $(100a+10b+c)$. It is evident that this may be written $\Sigma(10r)$ $(100a+10b+c)+\Sigma(s)$ $(100a+10b+c)$. Now it is possible that there will be more than one observation of the form $(100a+10b+c)$ for each individual value of s, as s_0, s_1, . . ., so that we may factor out the individual values of s. The expression $\Sigma(s)$ $(100a+10b+c)$ may then be written in the factored form $(s_0)\cdot\Sigma'(100a+10b+c)+(s_1)\cdot\Sigma'(100a+10b+c)+ \ldots +(s_9)\cdot\Sigma'(100a+10b+c)$ and the expression $\Sigma(10r)$ $(100a+10b+c)$ may be handled the same way. The factoring and recombining of these terms to give the original expression are illustrated in Table I.

TABLE I

Digit	Partial sum	Partial product-sum
s_0	$\Sigma'(100a+10b+c)$	$(s_0)\cdot\Sigma'(100a+10b+c)$
s_1	$\Sigma'(100a+10b+c)$	$(s_1)\cdot\Sigma'(100a+10b+c)$
.	.	.
.	.	.
.	.	.
s_9	$\Sigma'(100a+10b+c)$	$(s_9)\cdot\Sigma'(100a+10b+c)$
$10r_0$	$\Sigma'(100a+10b+c)$	$(10r_0)\cdot\Sigma'(100a+10b+c)$
$10r_1$	$\Sigma'(100a+10b+c)$	$(10r_1)\cdot\Sigma'(100a+10b+c)$
.	.	.
.	.	.
.	.	.
$10r_9$	$\Sigma'(100a+10b+c)$	$(10r_9)\cdot\Sigma'(100a+10b+c)$
Total product-sum $=\Sigma(10r+s)$ $(100a+10b+c)$		

The above process would not be very practical were it not for punch cards and sorting and tabulating machinery. In the illustration

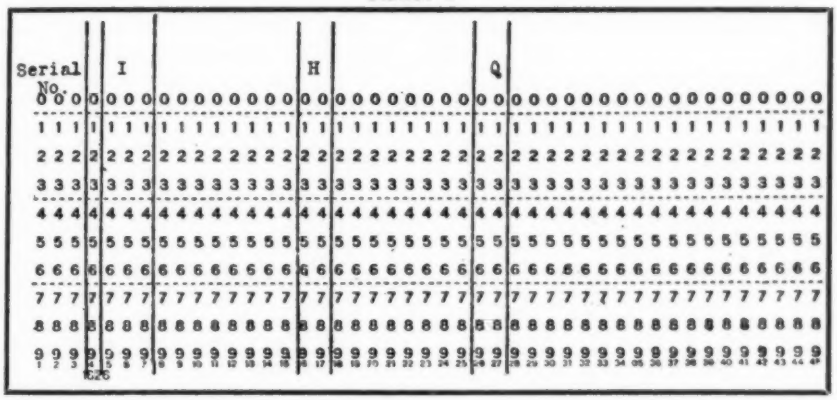
[1] "I" is the intelligence score, "H" is the high school average, and "Q" is the first quarter average of Iowa State College freshmen from the class which entered in the fall of 1926.

the necessary information was punched on cards as indicated in Chart I.

Those observations of the form $(100a+10b+c)$ were punched in columns 5, 6, and 7, respectively; those of the form $(10r+s)$ in columns 16 and 17; and those of the form $(10x+y)$ in columns 26 and 27. Information was punched in the other columns up to and including the 29th but for the present illustration only those groups of columns headed I, H, and Q were used. The numbers 1 to 492 were punched serially in columns 1, 2, and 3 for identification and to assist in verifying the punching.

After the cards were punched and verified they were put in the sorter

CHART I



and sorted on column 27, that is, the various values of y $(y_0, y_1, \ldots, y_9)$ were factored out. The cards were then placed in the tabulating and listing machine and by controlling on column 27, the partial sums of Q, H, and I were obtained for each value of y. The process was then repeated for columns 26, 17, 16, 7, 6, and 5 in the order named. These partial sums are the factors of the form $\Sigma'(100a+10b+c)$ in Table I which we will now indicate as $\Sigma'Q$, $\Sigma'H$, $\Sigma'I$. Tables II, III, IV, and V were made, except for the headings, directly by the machine.

It is a comparatively short and simple process to secure from the tables the necessary sums of squares and of cross-products. To secure the sum of the squares of the I observations on the Monroe calculator, for instance, the successive values of $\Sigma'I$ are placed in the machine and multiplied by the corresponding value in the "I Sort" column without clearing the products from the machine after each multiplication. The sum of the squares of each of the other variables are found in the same way, that is, by multiplying successive values of the partial sum by the corresponding values in the "Sort" column. To find the sum of the

TABLE II

Q Sort	$\Sigma' Q$	$\Sigma' H$	$\Sigma' I$
y			
0	3240	3567	5928
1	3799	4215	7052
2	4954	5337	8506
3	3057	3377	5414
4	4390	4753	7797
5	3820	4160	6698
6	4148	4578	7552
7	3033	3375	5701
8	4338	4795	7728
9	4010	4370	7251
10x			
40	271	482	584
50	1015	1481	1943
60	3743	4743	6899
70	8734	9851	15239
80	22552	23529	40229
90	2474	2441	4733

TABLE III

H Sort	$\Sigma' Q$	$\Sigma' H$	$\Sigma' I$
s			
0	3727	4140	6610
1	3759	4107	7010
2	4895	5364	8710
3	4163	4506	7765
4	4031	4468	7595
5	4739	5125	8292
6	3657	4012	6562
7	3163	3537	5419
8	3810	4154	6971
9	2845	3114	4693
10r			
70	2220	2467	3741
80	24953	27351	43390
90	11616	12709	22496

TABLE IV

I Sort	$\Sigma' Q$	$\Sigma' H$	$\Sigma' I$
c			
0	4284	4665	7940
1	3381	3730	5993
2	4257	4648	7528
3	3794	4180	7034
4	4319	4664	7616
5	3734	4148	6840
6	3579	3889	6280
7	3956	4353	7080
8	3678	4122	6634
9	3807	4128	6682
10b			
00	3462	3837	5717
10	4146	4707	6885
20	4968	5490	8156
30	4477	4858	7738
40	4990	5473	9404
50	3386	3685	6401
60	4248	4522	8133
70	3449	3753	6781
80	3213	3500	6167
90	2450	2702	4245
100a			
000	3213	3776	3904
100	33424	36495	60231
200	2152	2256	5492

TABLE V

Number of observations	$\Sigma' Q$	$\Sigma' H$	$\Sigma' I$
492	38789	42527	69627

cross-products of the I and H observations, ΣIH, the successive values of $\Sigma' H$ are multiplied by the corresponding values in the "I Sort" column which entails 20 multiplications in the illustration, or the values of $\Sigma' I$ may be multiplied by the corresponding values in the "H Sort" column, which entails but 12 multiplications, and which evidently produces the same result.

This method, which the author has called machine factoring, is used commercially under the name of digiting. Its application to inventories is explained in a booklet, *The Inventory Simplified*, published and distributed by The Tabulating Machine Company. The business applications are essentially the same as the one here outlined.

NOTES

MONTHLY ITEMS OF SECULAR TREND CORRESPONDING TO KNOWN YEARLY ITEMS

By C. H. Forsyth

The tendency seems to be growing to recognize the secular trend of many time series as parabolic and considerable work has been performed to fit parabolas to yearly values of many series. Those who have undertaken the work of eliminating such trend realize that the problem is quite different from that of eliminating linear trend. The purpose of this short paper is to present a method of eliminating the trend of any polynomial and of most any other form which has seemed appropriate up to this time. Since there is usually no great difficulty met in the particular problem of fitting an appropriate curve to the yearly values of a time series, the whole problem here resolves itself essentially to the determination of the corresponding monthly items from the graduated yearly items.

In the case of linear trend it happens that all we need to do is simply divide the coefficients of the equation, found to fit the yearly values, through by 12 to obtain the equation which gives the desired monthly values. Such a procedure does not work in any other case.

In the December, 1916, issue of this Journal the writer presented a formula for interpolating[1] individual values when the given values are groups of values. Strictly speaking, the formula is to be used to break up consecutive groups of values, of equal width, into smaller such groups whose widths are aliquot parts of those of the larger groups; in practice, these smaller groups can be and are almost invariably regarded as individual values.

In any extensive work calling for the use of the formula referred to, certain leading term and difference formulas are usually much more easily applied and, in addition, afford a very desirable check upon the work. These formulas have seen much service in work with vital statistics where the consistent tendency of the individuals enumerated,

[1] Those interested in a fairly extensive treatment of interpolation are referred to the writer's text *An Introduction to the Mathematical Analysis of Statistics*—the chapter on Interpolation—where considerable attention is given also to the type of interpolation treated in this paper; also to the Introduction to Pearson's *Tables for Statisticians and Biometricians*, and the chapters on interpolation in the *Textbook of the Institute of Actuaries*, Whitaker and Robinson's *Calculus of Observations*, Rietz's *Handbook of Mathematical Statistics*, etc.

somewhat hazy about their ages, to state their ages (or have their ages stated) in multiples of tens or fives, forces the Federal Government to publish the final results—population or mortality statistics—in the form of age groups.

The most useful form of these leading term and difference formulas, to second differences, is as follows:

$$T_1 = \frac{w_0}{t} + \frac{t+1}{2}\frac{\Delta w_0}{t^2} - \frac{t^2-1}{6}\frac{\Delta^2 w_0}{t^3}$$
$$\delta = \quad 1 \quad " + 1 \quad "$$
$$\delta^2 = \qquad 1 \quad "$$

Thus, if we have three consecutive groups, each of t values, w_0, w_1 and w_2 and arrange them as follows,

$$\begin{array}{ccc} w_0 & & \\ & \Delta w_0 & \\ w_1 & & \Delta^2 w_0 \\ & \Delta w_1 & \\ w_2 & & \end{array}$$

the values of T_1, δ and δ^2, computed by means of the above leading difference formulas, are arranged in the form

$$\begin{array}{ccc} T_1 & & \\ & \delta & \\ T_2 & & \delta^2 \\ & - & \\ T_3 & & \delta^2 \\ & - & \\ T_4 & & \delta^2 \\ & - & \\ T_5 & & \delta^2 \\ & \text{etc.} & \end{array} \qquad \text{(A)}$$

and added accumulatively (exactly the reverse of differencing) to give the desired set of interpolated values T_1, T_2, T_3, etc. The leading term and difference formulas given above are to be used to break up the middle group of the three groups w_0, w_1 and w_2, or w_1. Hence, a check upon the work is to be obtained by adding up the appropriate set of t values, T_1, T_2, T_3, etc., and comparing the sum with w_1.

In order that the procedure may be perfectly clear we shall take the risk of appearing too verbose and carry the work through for a simple problem involving hypothetical populations given in five-year age-groups. Hence, for this case $t=5$. The problem is exactly analogous

to breaking up yearly values of secular trend into monthly values, except that in the latter case t would have the value 12.

Ages	*Populations*		
10–14	$w_0 = 16{,}324$		
		$\Delta w_0 = 422$	
15–19	$w_1 = 16{,}746$		$\Delta^2 w_0 = 64$
		$\Delta w_1 = 486$	
20–24	$w_2 = 17{,}232$		

Hence (remembering that $t=5$),

$$T_1 = 0.2(16{,}324) + 0.12(422) - 0.032(64) = 3313.392$$
$$\delta = 0.04(422) + 0.008(64) = 17.392$$
$$\delta^2 = 0.008(64) = 0.512$$

Arranging these values (in italics below) according to scheme (A) and adding accumulatively, we obtain

Ages			
15	*3313.392*		
		17.392	
16	3330.784		*0.512*
		17.904	
17	3348.688		*0.512*
		18.416	
18	3367.104		*0.512*
		18.928	
19	3386.032		
15–19	16746.000	$= w_1$	

The work was carried to more decimal places here than would be necessary in practice, simply to illustrate the possibilities of check. Much exasperation will probably be avoided by anyone trying the scheme if he notes carefully from the start that interpolations are made in the middle one of any three given groups.

To bring the application still closer home, the parabola $y = a + bx + cx^2$ was fitted to the following yearly values (production of pig iron in units of 1000 long tons) to give the graduated values (and their differences) below. The details of fitting the parabola need not interest us here and are omitted. The equation becomes $y = 2280 + 1039.5x - 15.1x^2$. The graduated values given below are to be obtained by substituting appropriate values of x in the equation.

It will be noted that an extra decimal place is carried. This is desirable in such work but the extra place would, of course, be rounded off in the final results.

Now, as an illustration, let us break up the yearly value, say 23315.5

Years	Original values	Graduated values	Differences	
			Δ	Δ²
1903	17426	17821.0	1159.3	−30.2
1904	16131	18980.3	1129.1	−30.2
1905	22587	20109.4	1098.9	etc.
1906	24791	21208.3	1068.7	
1907	25307	22277.0	1038.5	
1908	15620	23315.5	1008.3	
1909	25397	24323.8	978.1	
1910	26849	25301.9	947.9	
1911	23326	26249.8	917.7	
1912	29375	27167.5		

for year 1908. Referring to the formulas for the leading term and differences

$$T_1 = \frac{22277.0}{12} + \frac{13}{2}\cdot\frac{1038.5}{12^2} - \frac{143}{6}\cdot\frac{(-30.2)}{12^3} = 1903.7$$

$$\delta = \quad 1 \quad " \quad + 1 \quad " \quad = \quad 7.2$$

$$\delta^2 = \quad\quad\quad 1 \quad " \quad = -0.02$$

Arranging according to plan (A)

January	*1903.7*		
		7.2	
February	1910.9		*−.02*
		7.2	
March	1918.1		*−.02*
		7.2	
April	1925.3		etc.
		7.1	
May	1932.4		
		7.1	
June	1939.5		
		7.1	
July	1946.6		
		7.1	
August	1953.7		
		7.1	
September	1960.8		
		7.0	
October	1967.8		
		7.0	
November	1974.8		
		7.0	
December	1981.8		
	23315.4		

Considerable freedom was taken here with the difference "$\delta^2 = -0.02$" which is so small in this case that it could almost be ignored,

since we are interested only in any effect it may have upon the last place in our interpolations.

It will be found that the theory treated here leads to exactly the same results in the case of linear trend as are given by the procedure ordinarily followed and referred to earlier in the paper. We have illustrated the theory with data in their original form. The use of relatives would have no essential effect upon the applications.

AN ESTIMATE OF TOTAL VOLUME OF DEBITS TO INDIVIDUAL ACCOUNTS IN THE UNITED STATES[1]

By Morris A. Copeland

The Federal Reserve Board receives every week reports on the debits to individual demand and time deposit accounts for banks in about 260 centers. Roughly speaking, these reports represent the total volume of money payments arising out of all kinds of transactions, or would, if the reports were complete for the entire country.[2] But in some of the reporting centers there are a number of banks that make no reports, and there is a considerable volume of business in the centers from which no reports are received, so that the total of reported debits falls somewhat short of the total for the United States as a whole. Fairly complete figures on individual deposits for the entire country are available. On the basis of these and the ratios of debits to deposits in centers for which reports on both are available, it is possible to estimate fairly accurately the total debits to individual accounts for all banks in the United States. This total in 1927 probably reached the enormous sum of eight hundred and twenty-three billion dollars.

The method by which this figure is obtained may be briefly described. Two different bases have been employed: (1) ratios of debits to demand deposits (debits to demand accounts are easily nine-tenths of all debits) and (2) ratios of debits to total deposits. Information regarding demand deposits as of (approximately) June 30, 1926,[3] is now available for all banks which report debits in 224 centers. This makes possible a somewhat more accurate estimate than the one which was made by Dr. Carl Snyder [4] on the basis of demand deposits in national banks in some sixty odd cities. Figures for total deposits for all banks in each of 244 centers which report debits are given in the Rand McNally Bank Directory. These figures, also for June 30, 1926, afford a second basis of estimate. The two estimates for 1926 differ by less than one-third of one per cent, each being some fifty billion dollars, or seven per cent, larger than Snyder's estimate.

The turnover ratios (debits divided by deposits) vary both with the size of the city and from one part of the country to another. The country was therefore divided into ten sections, and a separate estimate

[1] The writer is indebted to the Heckscher Research Fund of Cornell University for financial assistance in the calculations.

[2] Withdrawals from time deposit accounts and cheques cashed should be subtracted, and payments made by hand-to-hand cash added. The net amount by which debits would need to be increased to give the total volume of money payments is probably less than 5 per cent.

[3] Unpublished data, furnished by courtesy of the Federal Reserve Board.

[4] This Journal, June, 1924, pp. 189–90.

made for each section. The method employed was a modification of that developed by Snyder. The centers in each district for which both deposit and debit figures were available were classified into five groups according to the total amount of deposits in each center. In the estimate using demand deposits Group A included centers having over $100,000,000 deposits; Group B centers having $50,000,000 to $100,000,000 of deposits; Group C $25,000,000 to $50,000,000; Group D $10,000,000 to $25,000,000, and Group E less than $10,000,000. A double cumulative curve of debits and deposits was plotted, showing debits and deposits in Group A; debits and deposits in Groups A and B; debits and deposits in Groups A, B and C; etc. The total of debits for the district was estimated from the known total of deposits by extrapolating on a free-hand curve through these five plotted points.

The estimate on the basis of total deposits was made by the same method as that on the basis of demand deposits except that in classifying the reporting centers according to total deposits, the class limits were increased to $20,000,000, $50,000,000, $100,000,000 and $200,000,000. In effect this method amounts to calculating demand deposits not included in the banks for which reports on both deposits and debits are available; and applying to this a turnover ratio, estimated from the ratios for the five groups of reporting centers. Both in the case of the demand deposit method and in that of the time deposit method, the turnover ratios vary from large to small centers in so irregular a fashion in many of the districts that it is impossible to estimate accurately the ratio to apply to deposits of non-reporting banks. But fortunately in most cases the proportion of these deposits to the total for the district is so small that a wide margin of error in the assumed turnover would make only a small one in the debits estimate.

District	Estimate based on demand deposits	Estimate based on total deposits	Final estimate
Me., N. H., Mass., R. I., Conn., Vt.	37.5	38.3	37.9
N. Y., N. J.	*	364.0	364.0
Pa., Del., Ohio	72.0	72.0	72.0
Ky., Mo.	21.5	21.9	21.7
Wis., Ind., Ill., Mich.	88.5	91.2	89.8
Md., Va., W. Va., D. C., N. C., S. C.	24.4	27.0	25.7
Ga., Fla., Ala., Tenn., Miss., La., Tex., Okla., Ark.	51.2	51.4	51.3
Minn., Ia., N. Dak., S. Dak., Neb., Kan.	38.4	34.2	36.3
Mont., Colo., Wyo., N. Mex., Ariz., Utah, Ida., Nev.	8.9	8.9	8.9
Wash., Ore., Calif.	43.9	40.2	42.0
Total	....	749.1	749.6

* Figures on demand deposits were not available for a sufficient number of cities in the district to afford a satisfactory basis for an estimate.

An error of 33⅓ per cent in the same direction in the turnover ratios for the centers for which reports are not available for each of the ten districts would involve an error of less than 5 per cent in the estimated total debits for the United States, on the total deposit basis of estimate, and a greater error than this is very unlikely.

The estimates are shown in the preceding table.

Estimates for years other than 1926 have been made by the method employed by Snyder, the debits for 140 identical centers for each year were multiplied by the ratio $\frac{411}{269}$ and New York debits added. Four hundred and eleven billion dollars represent total debits outside New York City for 1926 and $269,000,000,000 debits in 140 centers. These estimates follow:

Year	(1) Debits for 140 cities Actual data	(2) Estimate for U. S. outside of N. Y. City*	(3) Actual debits for N. Y. City	(4) Estimate for entire U. S.
1919.....	211	322	244	566
1920.....	242	370	241	611
1921.....	192	293	207	500
1922.....	200	305	240	545
1923.....	225	344	238	582
1924.....	228	348	264	612
1925.....	256	391	313	704
1926.....	269	411	339	750
1927.....	282	431	392	823

* (1) multiplied by $\frac{411}{269}$.

It appears from this table that about 80 per cent of all United States debits are included in the 141 centers for which comparable reports are available over a period of years. In 1926 there were 250 odd centers reporting debits representing about 85½ per cent of the total for the entire United States.

MILITARY LOSSES IN THE WORLD WAR

By Walter F. Willcox

Professor Hersch of the University of Geneva has examined recently the losses of life direct and indirect caused by the World War.[1] His study is later, more detailed, and based on fuller sources than my "preliminary survey" printed in this JOURNAL[2] and his results, therefore, at most points, where they differ should be regarded as superseding mine. For two countries, however, the United States and the British Empire, I prefer my own figures. For the United States he accepted the War Department estimate of 1920 apparently overlooking the fact that in 1922 the Department increased its estimate by 6,000 and the further fact that neither estimate included losses of life in the American navy during the War. For the British Empire we differ mainly because I added first the 73,000 lives lost in the British navy and secondly, of the 247,000 reported "missing" from the British army, a special class of 101,000 "presumed dead." If Professor Hersch intends, as I hope he does, to bring together later into a book the two articles which have appeared and one or more others needed to complete his survey, it might be well for him to review his evidence about these two countries.

The figures in my table showed losses according to the War Department estimate of 8,558,000 and according to my revision of its figures of 9,460,000. My general conclusion was stated as follows:

> In view of the general tendency of such estimates to increase with the increase of knowledge, of the likelihood that the Russian military losses are seriously understated even by the larger of the two figures, of the fact that the German figures do not include any naval losses, and of the probability, very near a certainty, that among the 7,750,000 "prisoners and missing" and the 21,200,000 "wounded casualties" in the complete War Department table there must have been hundreds of thousands, if not millions, who died of wounds or disease resulting directly from the War, I am convinced that 10,000,000 is a conservative estimate of the number who died as a direct result of their military or naval service.

Professor Hersch's conclusion is that the War carried off in round numbers thirteen million mobilized men and I suppose he would agree with me in thinking that this is more likely to be an underestimate than an overestimate.

The main difference between us is over Russia. The War Depart-

[1] L. Hersch, "La Mortalité Causée par la Guerre Mondiale," *Metron*, Vol. V, Part I (1925), pp. 88–133 and Vol. VII, Part I (1927), pp. 3–82.

[2] W. F. Willcox, "Population and the World War: A Preliminary Survey," this JOURNAL, Vol. 18 (1923), pp. 699–712.

ment estimate of 1,700,000 applies evidently to the losses before the Russian revolution of 1917. My estimate of 2,500,000 derived from a statement of the Director of Russian statistics was thought to be a conservative figure. Professor Hersch raises it to 5,350,000 and I accept his evidence and result. This accounts for more than nine-tenths of the excess in his figures over mine. In several cases like Belgium, Serbia, and Rumania, I had to choose between two widely different totals coming from the two sides in the War. Thus the War Department reported, probably from a Belgian source, 14,000 and a German statistical authority 115,000 deaths of mobilized Belgians. Professor Hersch fixes on 40,000 and I agree.

At the end of his earlier paper he gives a table which affords a rough measure of the effect of the War in directly destroying lives. This he does by computing the per cent of the men between 15 and 49 years of age in each country before the War who lost their lives while in the army or navy. I have rearranged his table so as to place the countries in the order of decreasing proportional loss as so measured.

APPROXIMATE LOSSES, ABSOLUTE AND RELATIVE, IN THE ARMIES OF DIFFERENT COUNTRIES DURING THE GREAT WAR

Country	Males 15 to 49 years of age before the War (in thousands)	Military losses during the War (in thousands)	
		Total	Per cent
Serbia and Montenegro	1,216 *	325	26.7
Soviet Russia	32,117	5,000	15.6
Turkey	3,303 *	500	15.1
Rumania	1,809 *	250	13.8
France	9,981	1,320	13.2
Germany	16,316	2,000	12.3
Bulgaria	989	100	10.1
Austria-Hungary	12,176	1,200	9.9
Italy	7,767	700	9.0
Greece	1,384 *	100	7.2
Scotland	1,193	83	7.0
England and Wales	9,252	641	6.9
Former Russian Poland	4,128	250	6.1
Belgium	1,924	40	2.1
Ireland	1,094	20	1.8
Portugal	1,315	8	0.6
All European countries reported	108,794	12,637	11.6
United States of America	25,541	116	0.5

* One-fourth the total population at the outbreak of the War. Full notes explaining how the figures in the first two columns were reached may be found in the original article.

The main results of the table are to bring out clearly the very small percentage of losses of life in the United States, Portugal, Ireland, and Belgium and the devastating losses in Serbia, Soviet Russia, and Turkey. The average loss among the European belligerents was 23 times as great and the loss in Serbia 53 times as great as in the United States.

INTERIM REPORT OF THE PROCEEDINGS OF THE FIRST GENERAL ASSEMBLY OF THE INTERNATIONAL UNION FOR THE SCIENTIFIC INVESTIGATION OF POPULATION PROBLEMS[1]

The first General Assembly of the International Union for the Scientific Investigation of Population Problems was called to order at 10 a.m., July 4, 1928, by the Chairman of the Provisional Committee of Organization. This and the following sessions were held at the Musée Social in Paris.

The following delegates were in attendance at the sessions:

Belgium: Le Père Valère Fallon, Eugène Dupréel.
Denmark: Sören Hansen.
France: Léon Bernard, Caullery, Georges Risler, Lucien March, André Siegfried, Jordan, Huber, Landry.
Great Britain: Sir Bernard Mallet, F. A. E. Crew, B. Malinowski, G. Pitt-Rivers.
Greece: Stavros Zurukzoglu-Kunz.
Holland: H. W. Methorst, G. P. Frets, Mlle. M. A. van Herwerden.
Italy: Corrado Gini, Franco Savorgnan, Marcello Boldrini, Carlo Valenziani, Vincenzo Castrilli.
South America: J. H. de Paula Souza.
Spain: Severino Aznar, Joaquin Espinosa.
Sweden: K. A. Edin, Cösta Bagge.
Switzerland: W. E. Rappard, O. Schlaginhaufen.
United States of America: Raymond Pearl, E. M. East, Robert E. Chaddock, Robert S. Lynd.

Professor Léon Bernard, Chairman of the National Committee of France, was then elected Temporary Chairman, and called for the report of the Provisional Committee of Organization. This report was presented by Dr. Raymond Pearl, who stated that at the last session of the World Population Conference in Geneva, on September 3, 1927, there were passed the following resolutions:

> The World Population Conference resolves that a permanent international organization be created for the object of studying population problems in a strictly scientific spirit.
>
> A Provisional Committee is hereby authorized to prepare the Constitution for this organization.

In accordance with these resolutions the following Provisional Committee for the organization of an International Union for the Scientific

[1] This paper is a brief digest of the essential points of the work accomplished at the Paris meeting. The detailed and definitive minutes of the proceedings will be issued (to members of the Union) at a later date.

Investigation of Population Problems was then appointed, with power to coöpt additional members until the total number of the committee was fifteen:

	Representing
Sir Bernard Mallet	*United Kingdom of Great Britain*
Professor F. A. E. Crew	
Professor E. M. East	*United States of America*
Professor Raymond Pearl	
Professor Léon Bernard	*France*
Professor Erwin Baur	*Germany*
Professor Corrado Gini	*Italy*
Professor W. Rappard	*Switzerland*
Professor A. Mahaim	*Belgium*

Under the power granted for coöptation the following persons have been added to complete the committee:

	Representing
Professor Severino Aznar	*Spain*
Professor Kiyo Sue Inui	*Japan*
Professor H. Lundberg	*Scandinavia*
Professor J. H. de Paula Souza	*South America*
Dr. Charles H. Wickens	*British Dominions*
Professor V. Bunak	*Russia*

The committee organized at Geneva and appointed Professor Raymond Pearl Chairman, and Professor F. A. E. Crew Secretary. Since last September this committee has been actively engaged in the preliminary work of the organization of the union, with the financial and moral aid and support of the National Research Council and the Social Science Research Council, acting in coöperation.

The Provisional Committee, after the preliminary study and work throughout the academic year just past, met in Paris on July 2 and 3, 1928, and completed the final draft of the Statutes of the Union. At this meeting there were present all the members of the Provisional Committee of Organization *except* Professor Erwin Baur, Professor Inui, Professor Lundberg, Dr. Wickens and Professor Bunak. Professor Lundberg designated as his official substitute Dr. Sören Hansen of Denmark, who sat with the Provisional Committee, as did also, by invitation, Dr. Methorst, Chairman of the National Committee of Holland.

The Provisional Committee of Organization presented as its report the following statutes, which were, after discussion, unanimously adopted by the General Assembly on July 5, 1928.

STATUTES OF THE INTERNATIONAL UNION

I—Objects of the Union and Conditions of Admission

1. The purpose of the Union is to develop scientific studies pertaining to the problems of population, and particularly:

(a) To initiate and organize researches which depend upon international coöperation, to provide for the scientific discussion of the results of such researches, and to publish them without duplicating the publications of existing international statistical agencies.

(b) To facilitate the establishment of common standards for the collection, tabulation and analysis of data regarding human populations, including not only demographic, but also agricultural, economic, sociologic, and biologic data in the broadest sense.

(c) To serve as a clearing house for the interchange of information about population, for the purpose of facilitating researches.

(d) To coöperate to the fullest extent with other organizations of a scientific character having similar objects.

(e) The Union confines itself solely to scientific investigation in the strict sense, and refuses either to enter upon religious, moral, or political discussion, or as a Union to support a policy regarding population, of any sort whatever, particularly in the direction either of increased or of diminished birth-rates.

2. The admission of national groups to the Union shall be by two-thirds vote of the General Assembly. The national groups of the following countries shall be considered members without further validation, provided they form National Committees and elect delegates to the first or second meetings of the General Assembly hereinafter provided for: Argentine, Australia, Belgium, Brazil, Canada, Checkoslovakia, Denmark, France, Germany, Great Britain, Greece, Holland, Italy, Japan, Norway, Poland, Russia, Spain, Sweden, Switzerland, United States of America.

II—National Organizations

3. A National Committee, Society, Council, Association, or similar responsible organization shall be formed in each of the countries represented in the Union. It shall be formed under the responsibility either of a person entrusted with that duty by the authorities of the Union, or of a scientific academy of the country concerned, or of its national research councils, or of some other scientific institution or association of such institutions. In the event that there exist in any country several organizations fulfilling the required conditions the choice shall rest with the Executive Committee of the Union.

4. The functions of the National Committees shall be to promote and coördinate in their respective countries the investigation of the various scientific questions pertaining to the population, more especially in relation to their international needs. They shall be empowered, either singly or jointly with other National Committees, to propose questions falling within the purview of the Union for discussion by the Union.

Each National Committee shall nominate delegates to represent it at the meetings of the General Assembly of the Union, the number of such delegates never to exceed in number twice the number of votes allowed to the country it represents.

III—Administration of the Union

5. The affairs of the Union shall be directed by

(a) The General Assembly
(b) The Council of Delegates
(c) The Executive Committee

The final authority in the affairs of the Union shall be the General Assembly of the delegates.

6. The Executive Committee of the Union shall consist of a President, three Vice-Presidents, one of whom shall be also the Honorary Treasurer, who shall be elected by the General Assembly and hold office for six years, but a half of the officers shall be replaced every three years. At the outset it shall be decided by lot which of the officers initially elected shall retire at the second General Assembly of the Union.

The General Secretary shall be appointed by the Executive Committee of the Union with the approval of the Council.

7. The Council of delegates consists of one representative of each National Committee.

It meets at least once a year. It considers the Annual Report on the affairs of the Union presented by the General Secretary. It approves the accounts, examines the budgets and supervises the execution of the decisions of the General Assembly.

8. Retiring members of the Executive Committee are reëligible. The Council of Delegates shall have power to fill any vacancy occurring among members of the Executive Committee. Any person so appointed shall hold office until the next General Assembly, which shall then proceed to an election. The member thus elected shall complete the term of office of him whose place he takes. The person appointed or elected to fill such vacancy should preferably be chosen from the same country as the person whose office has become vacant.

9. There shall be a Central Administrative Office. The members of

its staff shall primarily be chosen for their technical qualifications and shall consist, so far as possible, of persons of different nationalities.

This office, under the direction of the General Secretary, shall conduct the correspondence, preserve the archives, prepare and issue the publications sanctioned by the General Assembly, and be responsible for the disbursement of funds, in accordance with the budget, and the decisions of the Executive Committee.

IV—Commissions

10. The General Assembly may appoint Commissions for the conduct of its work in the subjects decided upon. The Commissions shall present reports of their work.

11. The Chairman and the initial members of each Commission are elected by the General Assembly of the Union. They hold office for a period fixed by the General Assembly at the time of election, which period may, however, be extended by the General Assembly in case of necessity.

12. The Commissions may draw up their own by-laws, and may coöpt other members by a majority of two-thirds, with the approval of the Executive Committee. Such members need not necessarily be delegates but must be subjects of one of the countries adhering to the Union. All documents of interest to other members of the Union are transmitted to the National Committees through the Administrative Office of the Bureau.

13. With the sanction of the Executive Committee a Commission may issue its publications independently, and may entrust any part of its work to an institution or to individuals.

V—General Assembly

14. The Union shall, as a rule, hold an ordinary meeting of the General Assembly once every three years. The date and place of the meeting shall be determined by the preceding General Assembly. If, however, a General Assembly makes no decision as to place and date of the next meeting, these shall be determined by the Executive Committee, and communicated at least six months before the first day of the meeting to the National Committees. Any country in which the last or the next to the last meeting was held should preferably not be the place of the next meeting if there is a proposal of any other nation as a meeting place.

15. For special reasons, the President of the Union, with the consent of the Executive Committee, may summon an extraordinary meeting of the General Assembly. He must do so at the request of two-thirds of the votes of the National Committees, voting in accordance with their constitutional powers.

16. All members of the National Committees may attend the meetings of the General Assembly and take part in the discussion, but only delegates shall have power of voting. The President of the Union may, with the approval of the appropriate National Committees, invite other scientific men, who are not delegates but belong to one of the countries represented, or entitled to be represented in the Union, to attend a meeting of the General Assembly. Such invited guests may take part in the discussion without power of voting. Members of any of the Commissions referred to in Section IV who are not delegates shall have the right to attend those meetings of the General Assembly which deal with subjects referred to the Commission, without power of voting.

17. The agenda of business to be transacted at a meeting shall be determined by the Executive Committee after communication with the National Committees, and circulated at least six months before the opening of the meeting. No question which has not been placed on the agenda shall be discussed, unless a proposal to that effect be approved by at least one-half of the votes of the National Committees represented at the General Assembly.

18. If, and when, two-thirds of the National Committees adhering to the Union demand it, there may be held an International Congress on Population open to the general public, in the administration of which the Executive Committee of this Union shall have final authority, under the same principles as for General Assemblies of the Union.

VI—Finance and Voting Power

19. The Executive Committee shall prepare the budget for each year intervening between two General Assemblies. The Council of Delegates shall examine this budget and the accounts for the preceding term. The Council of Delegates shall then prepare separate reports to be submitted to the General Assembly for approval, which, having considered these reports, shall fix the unit of contribution for the succeeding period.

20. The contributions due from the adhering countries and their corresponding voting powers are determined by the population of the country according to the following scale:

Population	*Maximum number of votes*	*Corresponding number of units of contribution*
Less than 5 millions	1	1
Between 5 and 10 millions	2	2
Between 10 and 15 millions	3	3
Between 15 and 20 millions	4	4
Over 20 millions	5	5

21. Each National Committee is at liberty to include in the population of its country the inhabitants of its colonies. Self-governing dominions (South Africa, Australia, Canada, New Zealand, Irish Free State, India, Dutch East Indies, Newfoundland) have separate voting power according to the above scale. The Protectorates have the same power and pay on the same scale.

22. The contributions to be levied during the interval between the first and second General Assemblies shall not exceed £10 sterling per annum per unit. Each National Committee shall be responsible for the contribution of the country concerned.

A National Committee three years in arrears of dues fixed by the General Assembly shall cease to be a member of the Union.

23. The income of the Union is to be devoted to

(a) Expenses of administration, including salaries of the secretariat, and travelling expenses of the members of the Executive Committee, in connection with the business of the Union.
(b) Expenses of Commissions.
(c) Cost of publications.
(d) Promotion of the objects of the Union generally.

24. In addition to income derived from the sources described in Article 20, the Union may receive donations or bequests and disburse them in accordance with Article 23, subject to any restrictions made by the donor, and accepted by the Executive Committee when such donated funds are accepted.

25. If any National Committee should withdraw from the Union, it resigns at the same time its rights to share in the assets of the Union.

26. At the General Assemblies no vote shall ever be taken as to the merits of any purely scientific conclusion. Decisions relating to the organization and conduct of the scientific work of the Union shall be made by a majority of votes cast by the delegates present.

In questions of administrative or financial character, the vote shall be taken by National Committees, each National Committee having the number of votes designated in Article 20. In case of doubt as to the category in which a question belongs, the Chairman of the meeting shall decide. Votes taken in Committees shall be counted by individuals.

When there is an equal division of votes, the Chairman shall, in all cases, have a casting vote.

27. In questions of an administrative or financial character a National Committee not represented at a meeting may forward its vote to the President in writing, and such vote shall be counted if received before the other votes are counted.

VII—Regulations

28. The General Assembly may draw up Regulations for the conduct of its business, the general duties to be assigned to the Executive Committee of the Union and all matters not provided for in the statutes.

(a) Each Commission may, in like manner, draw up its own regulations, which shall be approved by the Executive Committee before coming into force.

(b) No regulation shall be made, either by the General Assembly or by the Commissions, which violates the terms of the present statutes.

VIII—Amendments

29. The French text of these statutes shall be considered to be the sole text of reference.

French and English shall both be the official languages for the proceedings and publications of the Union.

In a Congress or General Assembly papers may be read in Italian, Spanish, or German.

30. No change shall be made in the terms of the present statutes except with the approval of two-thirds of the votes of the adhering National Committees.

THE COMMISSIONS

Regulations

1. The Bureau of each Commission includes: A Chairman—appointed by the General Assembly of the Union; a Vice-Chairman—appointed by the Commission. The Commission may also appoint a Secretary.

2. The duty of the Commission is: To prepare the program of researches, studies and experiments relating to their special subjects; to examine from time to time the results obtained; to prepare reports for submission to the General Assembly of the Union.

3. The National Committees shall coöperate in the execution of the work carried out in accordance with the program prepared by the Commissions in so far as they are able.

4. National Committees shall report the results of their work, together with their observations and suggestions, to the General Secretary of the Union, together with a copy of the essential documents, and likewise an abstract of the results obtained. This abstract shall be communicated to all National Committees, and also to the Chairman of all Commissions.

ORGANIZATION OF THE INTERNATIONAL UNION

A nominating committee, appointed by the chair, brought in the following recommendations for the officers of the Union.

President: Professor Raymond Pearl.

Vice-Presidents: Professor Corrado Gini, Professor Léon Bernard, Sir Bernard Mallet.

These persons were unanimously elected to the offices named. Sir Bernard Mallet was also elected Honorary Treasurer.

A general discussion of problems which might profitably be investigated by the International Union brought out the following suggestions:

Professor Aznar (Spain): The factors influencing differential reproductivity, according to geographical regions, races, occupations, etc.

Professor Gini (Italy):

a. Historical study of population.
b. Demography of primitive populations.
c. The formation and development of new races.
d. Race crossing.

Professor East (U. S. A.): A comprehensive list of possible research problems in the field of population, falling under five general heads as follows:

a. Anthropology and Sociology.
b. Biology and Medicine.
c. Demography.
d. Geography and Economics.
e. Psychology.

Professor Edin (Sweden):

a. Methods for the analysis of statistics relating to occupation and fertility, and occupation and mortality.
b. Methods for the analysis of family statistics.

M. Lucien March (France): Population in relation to food supply, and the factors involved in the latter.

Father Fallon (Belgium):

a. Population in general.
b. Food supply.
c. Migration (geographical distribution of population).

Dr. Hansen (Denmark):

a. The relation between the mortality during the first year of life to that in the first month of life. (This suggestion was subsequently withdrawn.)
b. Has the span of human life been altered in recent times, or has the mean duration of life merely been altered by the reduction of infant (and childhood) mortality?

The Chairman appointed a committee to report upon the formation of Commissions in the light of these suggestions.

This Committee recommended that at the outset only three Commissions be formed, to deal with the following subjects:

Commission I. Population and Food Supply.
Commission II. Differential Fertility, Fecundity and Sterility.
Commission III. Vital Statistics of Primitive Races.

This recommendation was unanimously adopted by the General Assembly, and these three Commissions were constituted with the following personnel, it being understood that, as provided for in the Statutes, these Commissions would be enlarged later by coöptation.

Commission I—Population and Food Supply

Chairman: E. M. East (U. S. A.)
Vice-Chairman: Lucien March (France)
Members: Sir Henry Rew (Great Britain), Professor Brizi (Italy), Albert Henry (Belgium), O. E. Baker (U. S. A.), Erwin Baur (Germany).

Commission II—Differential Fertility, Fecundity and Sterility

Chairman: F. A. E. Crew (Great Britain)
Vice-Chairman: Severino Aznar (Spain)
Members: Raymond Pearl (U. S. A.), W. F. Willcox (U. S. A.), E. Mahaim (Belgium), T. H. C. Stevenson (Great Britain), A. M. Carr-Saunders (Great Britain), Lucien March (France), Furster (France), Grotjann (Germany), Zahn (Germany), H. W. Methorst (Netherlands), Corrado Gini (Italy), Franco Savorgnan (Italy), S. Zurukzoglu-Kunz (Greece), K. A. Edin (Sweden).

Commission III—Vital Statistics of Primitive Races

Chairman: Corrado Gini (Italy)
Vice-Chairman: B. Malinowski (Great Britain)
Members: G. Pitt-Rivers (Great Britain), Schmidt (Austria), Roquette Pinto (Brazil), O. Schlaginhaufen (Switzerland), Ales Hrdlicka (U. S. A.).

It was resolved, on motion of Professor Rappard that: The chairmen of Commissions I and II are requested to invite the National Committees to prepare for them, each for his own country, a report on population and food supply, differential fertility, fecundity and sterility, and vital statistics of primitive races.

The duration of the Commissions was resolved by the General Assembly to be six years.

The seat of the Union, until the next General Assembly, was chosen to be Paris.

The question of adherence of the International Union for the Scientific Investigation of Population Problems to the International Research Council was discussed at length, and the following resolution adopted:

The General Assembly postpones its adherence to the International Research Council, in spite of the moral interest which attaches to this adherence, and it empowers its Executive Committee to take steps to bring about such adherence as soon as circumstances seem to be expedient.

It was voted by the General Assembly that the dues to this International Union should be at the rate of £10 sterling per annum per unit.

There was a general discussion of the finances of this Union, and the hope was expressed that funds might be obtained to secure the adequate prosecution of the Union's work for the next five years, with the general understanding that whatever part of such funds might be raised in the United States must ultimately be approximately equalled by contributions from other countries of the world as a whole.

Brief informal reports were presented regarding the progress which has been made in the organization of National Committees in the different countries as follows:

Italy (Professor Gini)
France (Professor Bernard)
United States of America (Mr. Lynd)
Great Britain (Sir Bernard Mallet)
Holland (Dr. Methorst)
Spain (Professor Aznar)
Switzerland (Professor Rappard)
Belgium (Professor Dupréel)
Brazil and South America generally (Dr. de Paula Souza)
Sweden (Dr. Edin)
Greece (Dr. Zurukzoglu-Kunz)
Denmark (Dr. Sören Hansen)

The question of the publication of the results of the Union's work was informally discussed, and Professor F. Stuart Chapin presented to the General Assembly a brief account of the forthcoming periodical, *Social Science Abstracts.*

Professor Bernard urged the desirability of some form of publication by which the members of the Union might be kept currently informed of its activities. It was generally agreed, without formal action, that while it was too early to determine the final form which the publications of the Union might take, there was certainly a need from the start for the prompt dissemination of information to members, and that therefore some such bulletin as that suggested by Professor Bernard should be started as soon as funds would permit.

Sir Bernard Mallet moved a vote of thanks, which was carried by acclamation, to Professor Léon Bernard and the French National Com-

mittee for their hospitality and their work in organizing the meeting, and to M. Risler for offering the Musée Social as a meeting place. Professor Boldrini moved a vote of thanks, which was carried unanimously, to Professor Raymond Pearl for his work in organizing the International Union.

The General Assembly adjourned at 12 o'clock noon, on July 6, 1928.

STATISTICS OF THE INSURANCE BUSINESS

On Friday, May 18, the Association held a dinner meeting at the Aldine Club, New York City. The topic of the evening was "Statistics of the Insurance Business."

William C. Crawford, Insurance Editor of the *Journal of Commerce*, was the presiding officer. Papers were offered by Henry E. Niles, Assistant Manager of the Life Insurance Sales Research Bureau, Hartford, Connecticut; William Leslie, General Manager of the National Council on Compensation Insurance, New York City; Joseph Raywid, President, Underwriters Statistical Bureau, New York City; Hugh Thompson, Assistant Secretary, Queen Insurance Company, New York City, and President of the Insurance Accountants Association; and Richard C. Rice of J. K. Rice, Jr. and Company, New York City.

Mr. Niles discussed some of the problems encountered in the collection and use of life insurance sales data. He said that the purpose of collecting and distributing consolidated sales data from the 116 member companies in the Bureau was to throw light upon four major sales management problems in the life insurance business. The facts were used first to stimulate the sale of more and better life insurance; then to study and, if possible, prevent the unnecessary termination of life insurance policies; to reduce unit costs of life insurance sales; and lastly to build an adequate and efficient life insurance agency field force. Mr. Niles outlined the procedure in securing uniform data currently from the life insurance business. The prime difficulty is, and has been, to secure uniformity as to definition of the facts compiled and reported by the companies. In particular, this problem seemed to be most pressing in the field of termination or cancellation statistics.

Mr. Leslie outlined the problem of securing workmen's compensation data for use in rate making and in industrial accident prevention. He sketched the history of the several statistical plans in use in the workmen's compensation field since 1913. The evolution of statistical plans which led to the determination of adequate and responsible rates in workmen's compensation insurance, the procedures which led to the compilation of Schedule Z, the relative loss experience of small and large risks, and the proposed plan to report the current experience of workmen's compensation risks on the "unit-risk basis" were set forth in detail.

Mr. Raywid made a plea for more uniformity in the basic records of the casualty, fire and marine insurance business. He pointed out the advances which had been made during the past fifteen years in adopting modern tabulating procedures in the compilation of the loss experience and of the operating results of this group of companies. He said that if uniform basic record procedures could be adopted by the companies, tabulation costs could be cut, and experience could be combined so as to determine whether the underwriting policies followed by the companies were sound or whether they were in need of modification. He described briefly the service which was performed for the smaller companies by various tabulating bureaus.

Mr. Rice outlined the trend of insurance stock values, and said that through the activity of a number of brokerage firms, thousands of persons in all parts of

the country had become investors in the securities of insurance companies, and that this was one of the best safeguards which the insurance business had against hostile and unwarranted interference. Mr. Rice also ventured the prediction that within a few years the premium volume on aircraft insurance would outstrip the present tremendous volume in automobile insurance. Insurance would continue its progress, Mr. Rice said, in devising new forms of protection for old risks and in meeting the demand for new risk covers as they arise.

Mr. Thompson discussed the service of the statistician in the fire insurance business in securing data which would help the companies reduce the almost constant underwriting losses which had been suffered by that branch of the business during the past six or seven years. He said that the fire insurance companies reporting to the National Board of Fire Underwriters had operated at an underwriting deficit of $146,000,000 during the period 1921 to 1926, which was 4.4 per cent of the earned premiums over the period. This, he held, was a rather dismal exhibit for a business which played so large a part in the trade and commerce of the nation. The apparently prosperous condition of the fire companies was due, he said, to the market appreciation of security values, which for the time being offset underwriting losses. Securing adequate and equitable treatment for the fire insurance business is an economic problem of major importance. He urged a further study of fire insurance operating results according to the one-, three- and five-year types of contract. He said it was not entirely clear to fire insurance men whether the business was justified in offering protection for three years at 2½ times the annual rate, or for five years at 4 times the annual rate. The speaker also described the preparation of fire insurance statistics for more than 160 different risk classes, and urged further simplification of the tabulation plan. This has been done in two states, he said. Mr. Thompson also discussed some of the statistical problems in reporting fire insurance data for the determination of taxes. He felt that if uniform tax data practice could be established between the companies, a tremendous volume of correspondence now carried on each year, after tax returns are filed, might be done away with without curtailing the revenues due the states under the laws which tax fire insurance premiums.

Mr. George D. Moore, Assistant Secretary and Actuary of the Royal Indemnity Company, ably summarized the papers presented at the meeting.

In the course of the meeting, Mr. William Brieby paid a tribute to the memory of Mr. Joseph Hooker Woodward, a member of this Association, who died on Tuesday, May 15, 1928. Mr. Woodward was the senior member of the firm of Woodward, Fondiller & Ryan, Consulting Actuaries, and for many years was interested in the work of the American Statistical Association. Mr. Woodward was formerly president of the Casualty Actuarial Society, a Fellow of the Actuarial Society of America and of the American Institute of Actuaries, and a member of the American Mathematical Society.

It was the sense of the meeting that the Association could perform a distinct service to the insurance business if it maintained a standing committee on insurance statistics and if that committee were to promote further meetings of this character. It has been nearly twenty years since insurance statistics were discussed at any meeting of the Association. By offering facilities for the discussion

of statistical problems in the insurance business the Association would bring together a group of workers who today have no common meeting ground.

The Committee on Arrangements consisted of Arthur L. J. Smith, President of the Spectator Company, and dean of insurance publishers in the United States, Clarence Axman, Managing Editor of the *Eastern Underwriter*, G. Gordon Mackay of the *Insurance Index*, Thomas J. V. Cullen of the *Spectator* and E. W. Kopf of the Metropolitan Life Insurance Company.

MIGRATION OF INDUSTRY

The New England Council, through its Committee on Community Development, undertook last year the first study ever made to determine the extent of migration of industry in, and to and from New England. Lack of a definite answer previously permitted interests in other sections of the country to create the impression that New England was losing heavily industrially. This impression was strengthened by the fact that the loss of a few outstanding industries was widely advertised.

The study on the migration of industry in 1926 showed a net gain, after subtracting industries which failed or moved to other localities, of 224 industries and 10,998 employees. Twenty-three industries came from other sections to New England, while only eight moved away from New England. The study covered 197 communities.

The New England Council has been undertaking the study again for the year 1927. Complete reports have not been made for the State of Maine or the cities of Boston and Providence. However, the figures available, covering 173 communities, are comprehensive enough to show trends. In a statistical study of this kind the probability of error is so large that the figures can be considered accurately only as indications of general trends.

Of the 173 communities, 64 reported 209 new industries employing 12,169 workers. Forty-three communities reported losing, through failure or removal elsewhere, 92 industries employing 10,328 workers. In short, the net gain was 117 new industries and 1,841 employees.

In this connection, it may be noted that the average New England industry which failed or moved elsewhere in 1927 employed 112.7 persons as against 58.2 persons for the average new industry.

Another point of outstanding significance brought out by this study is that, whereas only 4 industries moved to points outside New England, 19 industries moved into New England from points outside. These figures do *not* include branch plants established.

Of the 209 new industries reported, 126 were new undertakings, 24 were branch plants, and 52 were removals from other communities. The type was not indicated in 7 cases. Approximately one-half the new industries were financed by local capital.

Of the 92 industries reported as lost by New England communities, 60 were due to failure.

This study on the migration of industry as it affected New England, developed certain definite conclusions:

1. That New England is maintaining a favorable balance in the exchange of industries with the rest of the United States.
2. That the development of new undertakings, and the acquisition of branch plants and industries from other communities leave a net gain in both number of plants and number of employees after taking into account those industries which have suspended operations or moved to other communities.
3. That the growth of industry in New England can best be promoted by local enterprises financed by local capital.
4. That the greater part of the migration of New England industry is from one New England community to another.
5. That it takes at least two new industries on the average to replace one that has failed or moved elsewhere.
6. That individual communities must devote more attention to the welfare of their established industries.

J. J. BABB

LEGISLATION FOR THE FIFTEENTH CENSUS

A bill providing for the next, or fifteenth, census, to be taken in 1930, was introduced at the last session of Congress and passed by the House of Representatives. In the Senate it was referred to the Committee on Commerce, in whose hands it remained at the close of the session.

The bill provides for a census of "population, agriculture, irrigation, drainage, distribution, and mines." Manufactures will be covered by the regular biennial census for the year 1929, and for this reason is not mentioned in the above list. The inquiry on distribution, which now appears on the census program for the first time, might be more specifically defined as a census of wholesale and retail trade. As the readers of this JOURNAL may recall, the Bureau of the Census, more or less as an experiment to determine the practicability of covering this subject, recently took a census of distribution in a number of selected cities. The results indicated that it would be practicable to include that subject in the next decennial census.

It is estimated that the census of 1930, with the scope defined in the present bill before Congress, including the current inquiries that will be carried on during the three-year census period, will cost not less than forty million dollars

The question of the date to which the census should relate has aroused a great deal of discussion. The census of 1920 was taken as of the first of January; but the difficulties encountered in taking a census of population in the winter months convinced the Bureau that the experiment should not be repeated. The census of 1910 was taken in April; but while the spring seems to be satisfactory for an enumeration of the population, there are some reasons why it is not the best time for a census of agriculture, the most important one arising out of the fact that nearly one-sixth of all farmers in the United States move from

one farm to another between the end of each crop season and the first of the following April.

It has been suggested that the censuses of population and agriculture might be separated and taken at different times; but that would involve a considerable increase in the total cost of the enumeration, and would have some other disadvantages.

In most respects the fall of the year would seem to be the best season for a general census; but when it is suggested that the fifteenth census be taken in the fall of 1929, there is objection to breaking away from the even decennial year. This objection may be largely sentimental, since the period from January 1, 1920, to November 1 (say), 1929, more nearly approximates a full decade than did the period between the thirteenth and fourteenth censuses (April 15, 1910, to January 1, 1920); but the objection is none the less vigorous. If, on the other hand, it is proposed to take the fifteenth census in the fall of 1930, and to establish a fall date in the decennial year for future censuses, there is the objection that once in twenty years the census-taking period will coincide with the period of a presidential election, which would be unfortunate for many reasons; and there is some question as to the constitutionality of a date so late in 1930.

The House Committee on the Census, after hearing extended arguments on all sides of this vexatious question, decided upon May 1 as the date for the next census.

In several particulars the present bill grants to the Director of the Census and the Secretary of Commerce a greater degree of authority and more freedom of action than previous census acts have done. For instance, the bill does not retain the provision of former census acts that the boundaries of supervisors' districts shall, so far as practicable, conform to the boundaries of congressional districts. The congressional districts, needless to say, are laid out without any regard to the requirements of the census, and the proviso that they should be used as supervisors' districts is not in the interests of good administration. In this matter of districting the country for the purposes of the enumeration the Bureau of the Census should have a free hand, as it will have, if the bill passes in its present form.

The rates of pay for supervisors and enumerators are not prescribed in this bill, as they have been for previous censuses. In 1920 the maximum rate of pay allowed by law was not sufficient to insure the obtaining of good enumerators and the census suffered in consequence.

The bill does not specify the questions that are to be asked in the census, but provides that the "number, form, and subdivision of the inquiries shall be determined by the Director of the Census with the approval of the Secretary of Commerce." Of course, the selection of questions is a matter of great importance. The new questions which various organizations or individuals want to have included in the next census make a long list. No final decisions have been made and the matter will be given very careful consideration. It is necessary to consider first of all the practicability of obtaining through a census the information called for by the proposed questions, and then the value of the information not only in itself but as compared with that obtainable by other questions which

may have been proposed or are already on the census schedules. There is grave danger of overloading the census with detail. We are already close to the limit—the limit to the amount and kind of information which can be obtained through a census, and the limit to the amount of detail that can be carried on the punch cards and successfully tabulated within the census period. If any number of new questions are to be introduced, some of the questions already on the schedule must be eliminated to make room for the new ones.

J. A. H.

MISCELLANEOUS NOTES

The Forthcoming Annual Meeting.—Announcement has been made by President Carl Snyder that Chicago has been chosen as the meeting place of the American Statistical Association as well as for the other various societies for their annual meetings for 1928. Headquarters of the American Statistical Association will be at the Stevens Hotel. Details of the program are not yet available.

United States Bureau of Labor Statistics.—A study of the settlements for accidents to American seamen has been completed. A statement of the principal findings of the study were published in the June issue of the *Monthly Labor Review*. The full report will be published in bulletin form.

The investigations of productivity of labor in merchant blast furnaces and in newspaper printing are ready for the press. The investigation of labor productivity in the loading and unloading of vessels has covered the Pacific and Gulf ports and is now being carried on at the Atlantic ports.

Field work on the wage investigations of the cotton manufacturing, woolen manufacturing, boots and shoes, men's clothing, and lumber industries is now under way. The regular annual survey of union wages is also under way. Similar wage studies for the electrical equipment industry and the manufacture of articles made of aluminum, brass, and copper have been completed, and summary reports are being published in the *Monthly Labor Review*.

The Washington Statistical Society.—At a meeting of the Washington Statistical Society on April 24, papers were presented on the subject of unemployment. Professor John R. Commons, University of Wisconsin, presided; Honorable Ethelbert Stewart, Commissioner of Labor Statistics, read a paper on the Department of Labor's estimate of the recent decline in employment, and Mr. Lawrence B. Mann, Department of Commerce, read one on "Recent Shifts in Occupational Distribution." These papers were discussed by Woodlief Thomas, Federal Reserve Board, and Isador Lubin, Institute of Economics.

Commissioner Stewart explained the methods of deriving the estimate of the Bureau of Labor Statistics of the recent decline in employment. He stated that the figure was not proposed to represent the volume of unemployment, but was simply an attempt to obtain an estimate of the decrease in the number of wage earners employed on the basis of the limited statistical information available. It was estimated that there were, in 1925, 25,220,000 persons working for salaries and wages for others in this country. Statistics of monthly trends of employment are collected for manufacturing industries and for railroads. These figures show a decline of 7.43 per cent from the average for 1925 to January 1928. Assuming that a similar decrease has occurred in all other branches of industry, this percentage applied to the estimate of total wage earners indicates that there was a decline of 1,874,000 workers from 1925 to January 1928.

Mr. Mann presented estimates of shifts that have taken place in the occupations of gainfully employed persons since 1919. These estimates show declines in number of persons engaged in production of over 1,500,000, in transportation of over 200,000, and in Government service of over 200,000. At the same time, however, there were increases in persons engaged in distribution of nearly 1,200,000, in domestic and personal service of nearly 700,000 and in the professions of over 200,000. Summarizing, the increases amounted to about 2,100,000, and the decreases to about 2,000,000,

making a net increase of about 100,000 persons. Mr. Mann pointed out, however, that the number of persons becoming available for employment in the same period amounted to between 2,500,000 and 3,000,000, from which should be subtracted an increase of a million in school enrollment beyond the growth which would be expected from the increase in total population. This gave a net change in number of persons available for employment of about 1,800,000, which would indicate a volume of unemployment of 1,700,000.

Mr. Thomas stated that Mr. Stewart's estimate, if taken as an estimate of unemployment, was based upon five assumptions: (1) that the indexes used indicated change in factory employment; (2) that the changes in other lines of industry were the same as those in manufacturing and in railroads; (3) that the decreases in wage earners were not absorbed by the increases in other lines; (4) that there was no unemployment in 1925, and (5) that there was no increase in the number of persons available for employment since 1925. In the first place the index of factory employment can be shown by comparison of census figures to have a downward bias of nearly 2 per cent a year, which would serve to exaggerate any estimate of decreased employment made on the basis of this index. Mr. Mann's studies indicate that the decline in employment which occurred in manufacturing and transportation did not occur in other lines, but in fact workers thus released were absorbed in other fields of endeavor, especially in distribution and personal service. It is likely that there was some unemployment in 1925, certainly in some lines, and for this reason 1923 is more satisfactory as a base from which to measure the decline in employment or the increase in unemployment. Making allowances for shifts in occupations and for the growth in school enrollment and for the increase in number of persons available for employment, Mr. Thomas estimated that the volume of unemployment in 1927, assuming full employment in 1923, was probably within a range of 700,000 and 2,000,000.

Mr. Lubin stated that one of the most important problems of the present employment situation was that of short time employment. It is probable that the number of workers on part-time work is larger than the number totally unemployed. The plight of those partially employed is difficult because although they suffer the disadvantages of reduced income they are not able to seek employment elsewhere because of the difficulty of giving up a certainty for an uncertainty and because of the possibility of more regular employment in their present positions. Statistics of partial employment are needed as well as those of unemployment.

At the annual business meeting, which followed the program, the following officers were elected for the year 1928: President, William M. Steuart, Director of the Bureau of the Census; Vice-President, E. A. Goldenweiser, Director of Research and Statistics, Federal Reserve Board; Secretary, Woodlief Thomas, Division of Research and Statistics, Federal Reserve Board.

Dinner Meeting of the Pittsburgh Chapter.—An unusually successful dinner meeting of the Pittsburgh Chapter of the Association was given on the night of Friday, May 18. The meeting was organized by Professor John H. Cover, of the University of Pittsburgh, District Secretary, with the assistance of Arthur W. Bass, of the Westinghouse Company, and Lester Bernstein, of the Philadelphia Company in Pittsburgh. Nearly three hundred representative business men, bankers, and statisticians of the district attended, and Mr. A. C. Robinson, of the Peoples Savings and Trust Company, took the chair. Present as speakers were Colonel Leonard Ayres, past President, and Carl Snyder, present President of the Association.

Mr. Snyder dealt especially with the rapid increase in our knowledge of industrial and trade phenomena and especially in our ability to measure and calculate the growth first of different lines of industry and now, latterly, as a whole. Some of these measures, like the growth of manufacturing, may now be carried back to 1840, physical production, building construction, and bank clearings to about 1870; and the results obtained from these are now supported by the returns from over 400,000 corporations, as to gross and net income, from 1917. These measures permit of a precision of statement and a certain prevision as to growth which only a few years ago were generally deemed chimerical. They have made possible a close measure of the fluctuations in trade and production, and have revealed how exaggerated were former beliefs as to the extent and duration of these fluctuations.

Colonel Ayres dwelt especially upon the qualifications for the successful application of statistical methods to business needs. Every craftsman must be master of his tools, but he must learn to use and adapt them to the particular job in hand, and this is especially true of the statistician. His success depends upon the ingenuity with which statistical methods are applied, rather than the inherent value of these methods. Colonel Ayres pointed his remarks with a number of vivid illustrations from his own experience in the War and abroad; as, for example, the successful transfer of some extremely scant data to the estimation of the proportion of different sizes of shoes needed in the army, and some similar problems. The great need, he felt, was adaptation to the immediate business in hand, and the ever-present value of mother wit. Mere training cannot make a successful statistician, any more than it can make a great engineer or an accomplished physician.

C. S.

The Cleveland Chapter.—On April 19 the Cleveland members of the American Statistical Association held their annual dinner meeting at the Hotel Winton, at which meeting the formal organization of the Cleveland Chapter took place. Under the provisions of the constitution adopted at this meeting the Chapter elected as its president Colonel Leonard P. Ayres of the Cleveland Trust Company, while Mr. J. D. M. Phillips of the National Association of Steel Furniture Manufacturers was elected vice-president. The Chapter also nominated Mr. H. B. Flinkers of the Federal Reserve Bank of Cleveland as district secretary.

There were three papers presented at this meeting: "An Index of General Business for the Fourth Federal Reserve District" by Mr. Flinkers; "Development of the Census Tract Method of Analysis" by Mr. H. W. Green of the Cleveland Health Council; and "More or Less Simple Statistical Methods" by Colonel Ayres.

Mr. Flinkers presented a local area index which differed materially in construction from the other indexes in use but which showed a very high degree of relationship when correlated with these same indexes. Among the series included were several non-production items, and to justify the addition of such series Mr. Flinkers stated that he thought that no index of *general business* could be adequate when based solely on production series, particularly when one considered the complexity of general business today.

The second paper was a continuation of the material presented by Mr. Green at an earlier meeting of the Cleveland group. By the census tract method of analysis any geographical area can be subdivided into a number of small units whose limits are fixed. Such division, Mr. Green pointed out, assists materially in studying the problems peculiar to that area. By means of a large number of charts he then showed such data as the geographical location of the several racial groups in Cleveland as

well as the geographical location of disease within the metropolitan area. Using also an illuminated board, Mr. Green was able to emphasize especially the changes which had occurred between census years.

Colonel Ayres' theme was a discussion of materials and methods and he cited a number of his experiences during the World War. The problem then as now was often a question of method but equally often it was only a question of materials. It is for the statistician to decide, therefore, between the two and to outline the attack on the problem presented. Colonel Ayres urged that the statistician should make it a point to inform himself thoroughly both as to the facts of the problem and as to the possible methods of analysis, for, he said, materials should not be permitted to outrun method nor should method be cast aside when materials demand precise measurement.

Following this meeting the Cleveland Chapter undertook a campaign for members with the result that the membership now totals about thirty, of whom twenty are active and ten associate members.

With the formal organization of the Cleveland Chapter there occurred also the formal organization of the Business Statistics Section of the Cleveland Chapter. This group first met in June, 1926, and has been meeting more or less regularly ever since. It now becomes definitely affiliated with the Cleveland Chapter as well as with the National Association.

There were five meetings of this section during 1927–1928 with an average attendance of about twenty. The chairman of this group is Mr. Harry A. Wembridge of the Joseph and Feiss Company, while Mr. Flinkers serves as secretary of this group also.

The October meeting was given over to a discussion of plans and the election of officers, while in November occurred the annual forecast meeting at which time each member of the group presented his forecast of business conditions for the coming year. The index used in forecasting was the Federal Reserve index of industrial production. From the individual estimates presented at this meeting a group composite was obtained and this composite was compared with the actual as the latter became available.

At the January meeting the speaker was Mr. D. C. Elliott of the Federal Reserve Bank of Cleveland and a member of the group who spoke on: "Some New Facts on the Gold Situation in the United States." The subject for discussion at the February meeting was: "The Elimination of Man-Power as a Factor in Industry," and we had as our guests and speakers Mr. B. C. Seiple of the Cleveland State-City Employment Bureau and Mr. Myron J. Jones, formerly personnel director at the B. F. Goodrich Company of Akron, Ohio. At the March meeting Mr. John W. Love of the *Cleveland Plain Dealer* and also a member of the group was the speaker, and his subject was: "The Industrial Development of the South."

The group was originally organized for the purpose of discussing business forecasts and we expect next year to undertake a more definite check-up of the forecasts made by the several statistical organizations in the country as well as an expansion of our own forecasting program.

Meeting of the San Francisco Chapter.—Following the dinner of the San Francisco Chapter on April 27, the group voted to accept the invitation of the Stable Money Association to meet with them and listen to an address by Dr. E. W. Kemmerer. The adjourned meeting was held on May 25 at the Bellevue Hotel, with thirty-four members and guests present.

The first speaker was Professor David Weeks of the Division of Agricultural Economics, University of California. He outlined the methods of analysis of time series as proposed by Rydner Frische, Professor of Economics and Statistics at Oslo, Norway.

The ordinary methods of analysis regard a time series as composed of trend, seasonal, cyclical fluctuations and accidental factors. Frische regards a series as a composite of many influences and endeavors to separate it into its many component parts. For instance, we may be studying an index of the price of meat animals. The cycles of cattle, hog, and sheep prices may have different durations. Beef prices will be influenced by hog and sheep prices. Substitution of other meat or vegetable products is important as well as the influence of the general business cycle and general level of commodity prices.

Frische uses a differential method and endeavors to separate such a complex series into its component parts. He takes the points where the data change direction and connects these points, takes the second differences, and multiplies by a constant.

In the normal point method of analysis the data are graphically smoothed by a three months moving average, with a second three months moving average taken of the first result. Each average has its terms weighted in a 1-3-1 ratio. Following this a curve is plotted showing second differences of the twice iterated moving averages. The next curve is obtained by taking second differences of the preceding curve and plotting. Succeeding curves follow this same principle. The curves cross the original data at points of flexion. After locating these points of flexion connect them by straight lines. This should be repeated until the final trend is a straight line.

Several trends are obtained during the progress of the analysis. The three months moving average is the lowest order of trend; then minor cycles are shown; then major swings. Some series, such as pig-iron production, break into five cyclical curves the last of which is a straight line. The method applies especially well to a series such as telephone traffic which has recurrent cyclical tendencies according to the hour of the day, day of the week, month of the year, and position of the year in the business cycle, as well as by long time rate of growth of the telephone habit.

In studying correlations the trend of one order in a series may correlate with the trend of a different order in the second series. Careful watch must be kept of the amplitude of the trends of each series and the length of period of the trend. If the number of cycles in the first curve per cycle in the second curve is small, the turning points must be watched carefully.

The second speaker was Dr. Holbrook Working of the Food Research Institute, Stanford University. He outlined short cut graphic methods of analysis which have been found useful in the work at the Institute.

The graphic method for correlation analysis makes use of the statistical concepts involved in the usual refined methods for correlation of time series, but substitutes a cruder and much less laborious technique, which is nevertheless adequate for many problems. The modifications of technique are (1) trends and regression lines are determined by the judgment of the statistician, aided by simple calculations, rather than by the method of least squares, and (2) necessary computations are performed graphically.

The method may be used (1) for preliminary analyses, to test hypotheses or to determine the nature of the problems involved. The accuracy of the method is almost always adequate and the economy of time and effort recommends it. (2) For studies not requiring the highest possible accuracy the method is suitable for the final as well as for the preliminary analysis. And (3) for practice in the use of correlation analysis the method is invaluable, both because of its economy of time and because of its "high visibility."

The procedure is first to plot the two principal series to be studied. If trends are pronounced or if computations are required, plot to logarithmic vertical scale. If one of the series is to be deflated, plot the deflator series and measure and plot (graphically)

the margin between the deflator series and the series to be deflated. Per capita figures and the like may be obtained similarly. The next step is to fit trends by eye to the series to be correlated—either the original or the adjusted series, as the case may require—using straight lines so far as possible. Take deviations from trends, graphically, and plot on scatter diagram. Then fit regression lines by eye. The approximate correlation coefficient may be calculated by the formula $r^2 = b_{12}\, b_{21}$ where b_{12} and b_{21} are the slopes of the two regression lines to their respective axes (the tangents of the angles). No account need be taken of the scales or of the standard deviations of the variables. The tendency with most workers will probably be to place the regression lines so as to result in a calculated correlation coefficient slightly too high. The residuals should be measured and plotted as deviations from the original trends of the variables. If necessary revise one or both trends. The final step is to investigate the correlation of the residuals with other variables.

Graphic methods may be used if within the range of error of the size of the sample. The probable error in economic data cannot be adequately measured by the usual formulae. For example, if we correlate a series with pig-iron production we may adequately measure the cycles over ten years by using six months averages. The data being limited to twenty items will have a high probable error. If we take daily figures with more than 3,000 items we secure a low probable error, yet the actual error involved is unchanged. The probable error is frequently much larger than is shown by calculation.

Meetings of the Los Angeles Chapter.—Meetings of the Los Angeles Chapter were held on March 5 and April 16, 1928. At the meeting on March 5, a paper was presented by Mr. J. L. Niceley on "Market Surveys," setting forth the procedure and methods required in these surveys. Mr. Charles C. Bowen presented a paper on the "Use of Statistics in Investment Banking," in which he emphasized particularly an analysis of the financial structure and problems of mortgage insurance corporations.

At the meeting held April 16, the Chapter was addressed by Mr. Floyd F. Burtchett of the University of California, at Los Angeles, who spoke on "Index Numbers." Professor Burtchett spoke of many uses of index numbers in studies of corporate problems, such as production costs, job requirements and savings effected by installation of new machinery. Mr. Llewellyn Bixby Smith, Assistant Treasurer of F. B. O. Studios, spoke on "Multiple Curvilinear Correlation," setting forth a graphic method of determining the degree of correlation of several variables. Mr. Smith used as a basis for his address data on temperatures in June, July and August and the yield of corn per acre in Kansas from 1890 to 1922.

Discussions of these papers and addresses followed the formal program at each meeting. The Los Angeles Chapter has twenty members and meetings are held informally at the University Club as dinner meetings. The present officers of the Chapter are President, George G. Ellis; Vice-President, A. T. Anderson; Secretary, Ira N. Frisbee; Treasurer, W. F. French; Editor, J. R. Douglas; Counsellors, H. E. Craig and J. L. Niceley.

Organization of The Boston Chapter.—At a meeting of members of the American Statistical Association, Boston District, held at the Boston City Club, on Friday evening, May 25, 1928, a Boston Chapter of the Association was formed, and the following officers were elected to serve for one year: President, Dr. Carroll W. Doten, Massachusetts Institute of Technology; Vice-President, Mr. Charles F. Gettemy, Assistant Federal Reserve Agent, Boston Federal Reserve Bank; Secretary-Treasurer, Mr. Roswell F. Phelps, Director of Statistics, Massachusetts Department of

Labor and Industries; Counsellors (to serve with the officers as a Board of Directors): Mr. K. B. Emerson, Business Research Department, United Shoe Machinery Company; Mr. Edwin B. Wilson, Harvard School of Public Health.

At this meeting, Professor Carroll W. Doten presided. There was an address by Professor Warren M. Persons of Harvard University on the subject, "Correlation of Weights and Relatives in the Construction of Index Numbers of Commodity Prices." There were 42 members and guests present, and much interest was manifested in the organization of the Boston Chapter of the Association and in the address by Professor Persons.

Meeting of the Chicago Chapter.—A meeting of the Chicago Chapter was held on Friday evening, May 4, at the Auditorium Hotel. The speaker of the evening was Professor Steven W. Gilman of the course in commerce of the University of Wisconsin.

Professor Gilman's subject was "Get the Range and Study the Target." He emphasized the necessity of keeping in mind that the target, the executive, must be studied when the statistician prepares his report. While technique is essential in the preparation of material for reporting, the document which goes forward must be stripped as far as possible of technical language, simple to the last degree, and straightforward. Forty-eight persons attended the dinner and a larger audience listened to the address. No further meetings will be held until fall, but the Program Committee is making definite plans for the programs at that time.

Meetings of the Austin Chapter.—On April 11 the Austin, Texas, members of the American Statistical Association and others associated with them held a meeting to organize a chapter of the Association. Twenty people attended this meeting, voted to organize, and then elected officers as follows: President, Edward L. Dodd; Vice-President, H. H. Schutz; Secretary, C. D. Simmons. Plans were made for four regular meetings a year and such special meetings as might be desirable. The officers were directed to draw up a constitution for consideration at the first regular meeting of the group in May.

On May 23 the first regular meeting was held, with twenty-six present. The topic was "The Measurement of Economic Relationships," with Dr. Ralph J. Watkins the main speaker. Professor Watkins' paper was principally a non-technical approach to the subject of multiple and partial correlation. A constitution was formally adopted and the Secretary directed to make application for affiliation as a chapter of the National Association.

Work of the Committee on Governmental Labor Statistics.—In June the Russell Sage Foundation made a grant of $7,500 to the Committee on Governmental Labor Statistics of the American Statistical Association to cover expenses of research in this field. In consequence the program of work of the Committee, of which Miss Mary van Kleeck is chairman, is being expanded. Mr. Bryce M. Stewart has been appointed Secretary and Miss Miriam West has been employed with the Committee during the summer.

Survey of Recent Economic Changes.—An investigation of recent economic changes in the United States is being made by the National Bureau of Economic Research, Inc., 474 West 24 Street, New York City, as a continuation under the same general auspices, and on a somewhat larger scale, of the survey entitled "Business Cycles and Unemployment," made by the National Bureau in 1922 for the Standing Committee of the President's (Harding's) Conference on Unemployment.

The earlier study was sponsored by a special committee under the chairmanship of Mr. Owen D. Young. A similar committee of the Conference stands sponsor for the present inquiry and is to draw its conclusions and practical recommendations from the fact-finding survey made by the National Bureau in coöperation with the best insight and knowledge of various qualified agencies and individuals. Among the coöperating organizations are the Social Science Research Council, the Bureau of Railway Economics, the Personnel Research Federation, the Institute of Economics, the American Engineering Council, and various Departments of the Government.

The funds for the fact-finding studies have been furnished by grants to the Conference from the Carnegie Corporation of New York and the Laura Spelman Rockefeller Memorial. The time schedule calls for publication in the second quarter of 1929. In addition to the research staff of the National Bureau, headed by Dr. Edwin F. Gay and Dr. Wesley C. Mitchell, directors of research, a large number of collaborating editors has been drawn from leaders in the field

Annual Meeting of the American Public Health Association.—The American Public Health Association will hold its 57th Annual Meeting at Chicago, Illinois, October 15–19, with headquarters at Hotel Stevens. Two other national health organizations, the American Child Health Association and the American Social Hygiene Association will meet jointly with the American Public Health Association.

There will be a total of 42 sessions, 31 of these will be meetings of sections of the Association including Health Officers, Industrial Hygiene, Food, Drugs and Nutrition, Laboratory, Public Health Engineering, Child Hygiene, Vital Statistics, Public Health Education and Public Health Nursing.

Uniformity in Accounting Terminology.—The American Institute of Accountants appointed a Special Committee on Terminology in 1921. Since then, many of the more important terms in accounting practice have been defined. The principles guiding the original committee were outlined in the *Journal of Accountancy* for July, 1922. In 1927, the Committee was reconstituted with Mr. Walter Mucklow, Jacksonville, Florida, as chairman. The Committee asks for coöperation in its work. Mr. Mucklow's address is 815 Barnett National Bank Building, Jacksonville, Florida. Definitions so far established will be found in the "Terminology Department" of the *Journal of Accountancy*.

Fire Losses.—The Committee on Statistics of the National Board of Fire Underwriters, Mr. Hart Darlington of New York, chairman, in a report dated May 24, 1928, states that after seven years of continuously mounting fire losses, the Actuarial Bureau estimates the destruction by fire in the United States during 1927 as $478,245,620. The report includes detailed statistics on fire losses from 412 American cities, the largest number yet represented in this annual compilation.

PERSONAL NOTES

Edmund E. Day has resigned from the University of Michigan and has accepted appointment as Director for the Social Sciences in the Rockefeller Foundation.

Roswell F. Phelps, the American Statistical Association District Secretary for Boston, is a candidate for the Republican nomination for State Auditor of Massachusetts. Mr. Phelps is now Director of the Division of Statistics in the Massachusetts State Department of Labor and Industries.

C. W. Parritt of Pittsburgh, Pennsylvania, has recently been appointed Statistician of the Department of Welfare of the State of Pennsylvania at Harrisburg. This position was recently held by Mr. Emil Frankel who is now Statistician of the New Jersey Department of Institutions and Agencies.

A. B. Cox, Director of the Bureau of Business Research, The University of Texas, is in Europe on leave of absence for the summer, engaged in cotton marketing research. During Dr. Cox's absence, Dr. Ralph J. Watkins, Statistician of the Bureau, is serving as Acting-Director.

In accordance with its established policy of enlisting the assistance of professors of economics on special problems during the summer vacations, the Dominion Bureau of Statistics is this summer availing itself of the services of Dr. A. Brady of the University of Toronto, and Messrs. J. Culliton of McGill University, Montreal, and D. B. Hurd of Brandon, the first two on the vexed question of "origins" and "destinations" of imports and exports, while Mr. Hurd is working on special studies in connection with the census of 1921.

COMMITTEE APPOINTMENTS, 1928

Committee on Nominations

E. Dana Durand, Chairman — Wesley C. Mitchell
Warren M. Persons

Committee on Program

Spurgeon Bell — A. J. Hettinger
Theodore H. Brown — John R. Riggleman
May Ayres Burgess — George Soule
Edgar Sydenstricker

Committee on Institutional Statistics (Additional Appointment)
Richard K. Conant

Following the precedent set by Ex-President Day, the Committee on Nominations has been selected from among the past presidents of the Association.

MEMBERS ADDED SINCE JUNE, 1928

Adams, Sidney J., University of Texas, Austin, Tex.
Beatty, John D., Secretary, Bureau of Recommendations, Carnegie Institute of Technology, Pittsburgh, Pa.
Bratt, Professor Elmer C., Statistics Department, University of Wisconsin, Madison, Wis.
Burtt, Dr. Harold E., Ohio State University, Columbus, Ohio.
Caffrey, Edward R., Statistician, Pot Devin Machine Company, 1221 West 38 Street, Brooklyn, N. Y.
Dean, Professor Prentice N., University of Pittsburgh, Pittsburgh, Pa.
Doyle, George A., Commercial Engineer, Bell Telephone Company of Pennsylvania, Pittsburgh, Pa.
Drake, Leonard A., Clark, Dodge and Company, 51 Wall Street, New York City.
Drisko, Benjamin B., 17 Gramercy Park, New York City.
Ettinger, Dr. Clayton J., University of Pittsburgh, Pittsburgh, Pa.
Finley, F. E., Bureau of Agricultural Economics, United States Department of Agriculture, Austin, Tex.

Frederickson, F. J., 60 Beaver Street, New York City.
Froggatt, Joseph, Jr., 74 Trinity Place, New York City.
Gile, Dr. Bueford M., College of Agriculture, Fayetteville, Ark.
Gusler, Gilbert, 231 South LaSalle Street, Chicago, Ill.
Ham, Elizabeth C., 285 St. Johns Place, Brooklyn, N. Y.
Hoffschmidt, Joseph J., Accounting Department, Celanese Corporation of America, 15 East 26 Street, New York City.
Hubbell, Walter S., School of Business, Columbia University, New York City.
Irwin, H. S., Grain Futures Administration, U. S. Department of Agriculture, Chicago, Ill.
Jenkins, B. L., Cashier, Old National Bank and Union Trust Company, Spokane, Wash.
Jones, Harry W., Actuarial Department, The Mutual Benefit Life Insurance Company, 300 Broadway, Newark, N. J.
Joy, Aryness, Division of Research and Statistics, Federal Reserve Board, Washington, D. C.
Kavanagh, M. J., Jr., Bureau of Business Research, University of Texas, Austin, Tex.
Kemper, Mark, Lumbermen's Mutual Casualty Company, Chicago, Ill.
Kenney, John F., University of Michigan, Ann Arbor, Mich.
King, K. P., Bureau of Census, Washington, D. C.
Kolesnikoff, Vladimir S., School of Business, Columbia University, New York City.
Lawrence, Lucile, Barnard College, New York City.
Lewis, George M., University of Texas, Austin, Tex.
Lord, Elsie E., American Telephone and Telegraph Company, 15 Dey Street, New York City.
Mann, Professor Albert Z., Director of Community Extension Service, Garrett Biblical Institute, Evanston, Ill.
McCann, John B., Statistician, Department of Public Works, Pittsburgh, Pa.
McDowell, Carroll R., Charity Organization Society, 105 East 22 Street, New York City.
Mizuno, Noboru, The Kyushu Branch of the Nippon Life Insurance Company, Hashiguchi Cho, Fukuoka City, Japan.
Mohlman, Dora K., East Harlem Nursing and Health Service, 354 East 116 Street, New York City.
O'Connor, James V., Brooklyn Edison Company, Pearl and Willoughby Streets, Brooklyn, N. Y.
Pett, Harris G., Division of Research and Statistics, Federal Reserve Bank, Chicago, Ill.
Ray, James E., Susquehanna Silk Mills, 149 Madison Avenue, New York City.
Robb, Richard A., 19 Seyton Avenue, Langside, Glasgow, Scotland.
Rochester, Anna, 85 Bedford Street, New York City.
Ross, Emerson, Statistical Department, Metropolitan Life Insurance Company, 1 Madison Avenue, New York City.
Spiegel, Julius, Bankers Loan and Trust Company of America, 516 Fifth Avenue, New York City.
Stein, Professor Lillian C., Department of Statistics, University of Pittsburgh, Pittsburgh, Pa.
Symonds, Dr. Percival M., Columbia University, New York City.
Thompson, Arthur E., Globe Indemnity Company, Newark, N. J.
Thompson, Walter, 15 East 41 Street, New York City.

Venus, Henry A., Actuarial Department, The Mutual Benefit Life Insurance Company, 300 Broadway, Newark, N. J.

Wilson, K. S., Statistical Department, Kaufmann's Department Store, Pittsburgh, Pa.

Winchester, Laurence S., Statistics Division, Duquesne Light Company, 435 Sixth Avenue, Pittsburgh, Pa.

Winters, Fred W., Bell Telephone Laboratories, 463 West Street, New York City.

REVIEWS

Frequency Curves and Correlation, by W. P. Elderton. Second Edition. London: C. & E. Layton. 1927. vii, 239 pp.

Twenty-two years ago the first edition of this book was prepared for the primary purpose of introducing certain new methods of higher statistics to the actuarial profession. The book has become a standard reference on the subject of the Pearsonian skew frequency curves, somewhat easier for the novice to grasp than the more elaborate memoirs in which Pearson developed his methods, but yet thorough and scholarly. The section on correlation was a real contribution to the understanding of a new subject in its day. The development and uses of correlation theory, however, have become so great that the treatment does not stand out in the same way as does the treatment of frequency curves. Other writers on educational, economic and mathematical statistics have investigated extensions of the theory which are not touched on here—such as correlation of time series, with special reference to determining the lag of one series as compared with another, the reliability coefficient, and the use of the coefficient in connection with psychological tests. This treatment of correlation, however, is valuable as an introduction to some of the main aspects of the subject, even if it can hardly be considered a complete treatise.

The changes in the new edition include the following:

(a) Revision of the key table summarizing the main features of the various Pearsonian frequency curves. The new table presents a clearer and more complete analysis.

(b) A discussion of uncommon frequency types, added in an appendum in 1917, and now transferred to the appropriate place in the text.

(c) Addition of a brief discussion of the Charlier system of curves in comparison with the Pearsonian curves.

(d) Expansion of the original 44 pages on correlation to 75 pages, including a new short chapter on partial correlation.

In view of the high scientific standing of the author, the reader is somewhat surprised to find a number of vague or inaccurate statements. For instance:

(a) "The following statistics . . . give the numbers of wives tabulated for the ages of mothers, and according to years since marriage" (page 91). The statement does not indicate what group of wives is under consideration nor whether the basis for assigning a particular wife to a particular year after marriage is her death, birth of a child, application for divorce, or status at a particular date.

(b) "We say . . . that a man marrying late in life usually takes a wife who is older than the wife of a man marrying earlier" (page 136). Presumably the author means to compare the ages *at marriage* of these wives, but he does not say so.

(c) The definition of correlation is: "Two measurable characteristics, A and B, are said to be correlated when, with different values x of A, we do not find the same value y of B equally likely to be associated" (page 137). In the opinion of the reviewer, the definition of a quantity which is to be measured should not be stated negatively, but in some way that suggests how to measure the extent to which the quantity is present in a given case.

These infelicities of expression merely show, perhaps, that "even Homer sometimes nods," but suggest that mathematical statisticians need to watch elementary matters of statistical exposition as carefully as everybody else needs to.

The prominence of this book as an expositor of frequency curve theory occasions regret that some of the most important questions in regard to the use of frequency curves were not given fuller treatment. Such questions include:

1. Why find an *equation* for a frequency distribution? Why not just plot, determine the median, mean and mode, and the various measures of dispersion, and make no further analysis? Within the past few weeks, as it happens, the reviewer ran into a problem which was solved by differentiating the equation of a frequency curve, but, in general, the equation seems to be used only as a device for graduation. Why should not other devices serve equally well for summarizing the distribution, comparing it with other distributions, or even for smoothing it?

2. Why use the Pearsonian frequency curves rather than the Charlier types? While Mr. Elderton devotes several pages to this topic, the question seems to deserve more complete treatment. Pearson and his disciples seem to be able to point to a somewhat more extended list of successful fittings. It is difficult to decide, however, whether this indicates a real superiority in the Pearsonian equations or merely greater numerical zeal on the part of their advocates.

3. Why find the constants in the equations of the curves by the method of moments rather than by the method of selected points or by adjusting the constants so that the proposed theoretical frequency curve will have the same area, mean, median, interquartile range or other statistical characteristics as the actual distribution?

While the book thus seems to fall short of perfection in certain points, there can be no doubt of its value as an essential reference book in any library that attempts to cover statistical theory. For this purpose, the new edition contains such substantial improvements that the 1906 edition can no longer be considered to fill the need for a detailed instruction manual on the methods to be used in fitting Pearsonian frequency curves and for a careful exposition of certain fundamental ideas in regard to correlation.

ROBERT W. BURGESS

Western Electric Company, Inc.

The Rate of Living, by Raymond Pearl. New York: Alfred A. Knopf. 1928. 185 pp.

The general tendency of the biologic sciences is to become more and more analytic, to study structure and function in ever finer detail. Dr. Pearl is in-

terested in obtaining a synthetic viewpoint and one which can be subjected to quantitative treatment. In this book he attempts to measure, as a whole, the life process itself. Moreover, it is not the vitality of the individual that is his ultimate objective, but that of populations. The book begins with an eloquent plea for recognition of the biology of groups as a "selbstandige Wissenschaft." The author calls attention to the remarkable dearth of quantitative data on that most important property of groups of organisms from the evolutionary viewpoint, their mortality rates, and describes his efforts to remedy this situation in at least one case by the study from all angles of the vital statistics of the fruit fly, Drosophila melanogaster.

It turns out that ordinary wild stocks of Drosophila have a survivorship curve (starting with the imago) which is curiously similar to that of a human population (starting from twelve years as the point of minimum death-rate) on comparing days of life of the former with years of the latter. Hasty generalization, however, is prevented by the discovery that one of the mutant varieties of Drosophila "vestigial wing" shows a markedly different curve. It also shows a mean length of life only one-third to one-half that of the wild stock. In man and wild type Drosophila, the death-rate rises only slowly up to two-thirds of the life span and then rises rapidly. In the vestigial winged flies there is an approach to a uniform death-rate. The difference between the two strains of flies is shown to segregate in typical Mendelian fashion as an element in a complex of character differences determined by a single unit of heredity. It is later shown that there are marked differences in mean duration of life in different wild strains of Drosophila and that these differences persist as long as studied (eighteen generations).

Various environmental conditions are also shown to have effects on the mortality curve. The author finds density (number of flies per bottle) to be among the more important of these, both in effect on the average and on the form of the curve.

The demonstration of genetic differences in duration of life leads to a discussion of the nature of that which is inherited. Various attempts at a definition of life are examined. The author reaches the conclusion that an organism is "an aggregation of matter which has the property of autonomously changing external sources of energy and non-living matter" into products of various sorts "by virtue of its pattern or organization." He defines vitality as "the degree of intensity of vital action," to be measured by functions which involve the "rate of energy transformation of the organism within limits in respect of time and space."

Up to this point (page 101), the reviewer was able to follow the author's viewpoint sympathetically on the whole and with a feeling of contact with stimulating ideas. In the remaining fifty pages of the text the stimulation continued but oppositely directed. A definition of inherent vitality about which all of the later discussion centers, seems to be the cause of his difficulty.

From the viewpoint of the organism as an *energy transformer* and of vitality as degree of intensity of energy transformation, it would seem that inherent vitality should be measured by the maximum quantity of energy which the organism is capable of transforming per unit of time under optimum conditions. This seems the only viewpoint in harmony with the author's discussion up to this point.

The time integral of this rate—the total transformation of energy during its lifetime, "the sum of energy stored plus energy set free," of which the organism is capable under optimum conditions, might, however, also be used if it is desired to include duration of life as an aspect of vitality which the author, in spite of his definition of vitality as a rate, evidently wishes to do. Theoretically, vitality might then be measured roughly either by total energy output (including storage) or by the total calories in the food digested, since these must be the same. The reviewer was all set for a discussion along these lines, when he was let down with the information that the proper conditions for measurement were those of complete starvation. "As a working hypothesis, inherent vitality is defined as the total capacity of the organism to perform vital actions in the complete absence of matter or energy of exogenous derivation."

This concept seems to have little in common with that preceding. At best, it can measure little more than the amount of energy stored in the previous history of the organism, complicated by the closeness with which the pattern approaches that of the "One Hoss Shay" in relation to the peculiar stresses of starvation. Since all that is inherited is present in the one cell stage, it might seem that this test should be applied there, but in practice, Pearl applies it much later: To Drosophilas which have passed through the larval and pupal stages, and to melon seeds in which the first leaves with their store of food material have already been developed.

The livestock breeder considers that a high degree of "vitality" is a first essential whether he is concerned with beef cows or dairy cows, draught horses or race horses, mutton sheep or wool sheep, etc. The capacity for transforming large quantities of food into either "energy stored or energy set free" is undoubtedly great in good individuals of any of these kinds of animals. It seems doubtful whether their respective capacities to remain alive in the complete absence of new supplies of energy to transform would accurately rank them as energy transformers.

To return to Pearl's results, he finds that starved wild Drosophilas live on the average forty-five hours instead of as many days. The form of the life curve on a percentile scale of life span is not much changed. Starved vestigial winged flies show virtually the same average length of life and the same form of life curve as the wild in contrast with the striking differences when well fed. The conclusion is that the inherent vitality of the two varieties is the same, but that "the potential capacity of vestigials in inherent vitality is able to come to only about one-third to one-half its complete expression" when well fed.

The studies are continued with melon seeds, allowed to sprout and grow in the absence of food and in complete darkness. The quickness of sprouting and the amount of growth in forty-eight hours following sprouting, are found to be closely correlated and are considered to be measures of inherent vitality. The regression of growth rate on time to sprouting, superimposed on the life curve of starved Drosophilas in such a way that it begins and ends at the same place, shows a certain similarity in form. "It is not without a certain impressiveness that two such totally independent pieces of research should under the influence of a synthesizing idea of much later date, yield such a concordant picture." The re-

viewer is not impressed. Aside from the seeming irrelevance of the comparison, there is an uneasy feeling that if duration of life were taken as the measure of inherent vitality of the melon seeds, as it is in the case of the Drosophilas, it might not agree with the measure based on rate of growth. It turns out, in fact, that just this is the case. The author proceeds in a later chapter to find the length of life of the starved melon plants and among other things, states that this shows a correlation of $-.643 \pm .057$ with rate of growth, *i. e.* if the latter accurately measures inherent vitality, *shortness* of life is a moderately good measure of the same thing.

If it be assumed that the hypothetical inherent vitality is equally correlated with rate of growth and *length* of life, it is easy to show that this correlation cannot exceed $+.42$. They are yardsticks with standard errors of estimate 90 per cent as large as the standard deviation of the thing they measure. The author takes pains to emphasize that in these experiments "length of life depends inversely on the rate of living" and is inclined to extend this conception beyond starvation conditions. Nevertheless, he continues to the end to consider both rate of growth and length of life (under starvation) as measures of inherent vitality whose agreement in form of distribution (in melons and Drosophilas respectively) is of significance.

In addition to the objections above, the reviewer is far from satisfied that either of these quantities measures anything inherent in the sense of inherited. While the author emphasizes the care taken to eliminate environmental variations during the period of starvation, it is not clear that the variation manifested during this period may not trace to environmental differences in the earlier periods of larval or embryonic growth. The disappearance under starvation of the effects of the known genetic difference between wild and vestigial flies, is not reassuring in this connection. In short, Dr. Pearl has described biometrically, certain phases of the reaction of melon seedlings and Drosophilas to starvation. It does not seem to the reviewer that it is illuminating to assign the rather ambitious term "inherent vitality" to any of these phases.

SEWALL WRIGHT

University of Chicago

Forecasting Business Conditions, by Charles O. Hardy and Garfield V. Cox. New York: The Macmillan Company. 1927. x, 434 pp.

Business Forecasting and Its Practical Application, by William Wallace. With an Introduction by W. T. Layton. London: Sir Isaac Pitman and Sons, Ltd. 1927. xiv, 117 pp.

These two books have quite similar purposes and points of view and go about their jobs in not dissimilar ways. Each is interested in business forecasting not as a science per se, or as an academic exercise, but as something with a "practical application." Each desires to bring home the practicability of business forecasting to the business men in the countries of which the respective authors are residents. Mr. Wallace's book, written for British consumption, tries to "put over" business forecasting even more aggressively than does the American

volume. At the same time one feels that Mr. Wallace is always more on the defensive than Messrs. Hardy and Cox. For this, the relative scarcity of British business statistics and the relatively slight amount of use made of such data in Britain as compared with their widespread availability and much greater use in this country, are doubtless chiefly responsible. Both works are permeated by a healthy and proper skepticism in regard to the utility of statistics when used without the aid of other facts in forming business judgments.

Nor do the similarities end here. By way of illustrating the sorts of jobs that others have done, both volumes give space to the work of the Babson, Brookmire and Harvard services, and reproduce typical charts of these services. Mr. Wallace's book also includes some of the charts of the London-Cambridge Economic Service, while the Hardy-Cox book, confining itself to this country, devotes space to the Moody and Standard Statistics services. The charts and theories of the various services are described in each book without critical comment; unless forsooth one might consider as a critical comment such a delightful remark by Mr. Wallace as the following, in reference to one of the services: "The Business curve, naturally perplexed by these movements (of an ancillary curve), ceased its downward fall in the last quarter of 1923. . . ." Both works devote a minimum of space to the subject of statistical technique.

Of the 314 pages comprising the body of the Hardy-Cox book (the last 120 pages are devoted to appendices), just over one-half are represented by Chapters XII to XXII, inclusive; each of which eleven chapters is devoted to some economic factor, such as agriculture, mineral production, or construction, or to a discussion of some type of index in the field of economic statistics, such as production, trade, wages, and prices. The chapters on the economic factors deal with these factors under one or more heads in varying degrees. Thus the chapter on agricultural production deals with the question as to the extent to which "business prosperity is closely dependent upon the size of the principal crops," in other words, as to the causative influence of agriculture on the business cycle. The chapter on the production of minerals, on the other hand, discusses very briefly, and it must be confessed quite inadequately, the effect of the business cycle on mineral production; in other words, causation in the opposite direction. The chapters on construction and railway transportation, as well as the chapter on banking transactions and the money market go much further into the subject of available indexes than the chapters on agriculture and minerals. In a five-page chapter on profits, the principal conclusion is that whether or not the profit theory of business cycles is correct, the data that would substantiate it are all too scarce and, "moreover, such data as we do have give little confirmation of the view that the shrinkage of profit margins is a major factor in causing the advent of depression." Finally, the chapter on the stock market, after indicating with little critical comment the principal stock price indexes generally available, gives a conventional discussion in a few pages of the why's and wherefore's of the movements in stock prices, both short-term and long-term. It concludes that neither the discounting propensities of the stock market, nor fluctuations in money rates explain fully timing and direction of stock market movements. The chapter also concludes that allowing for "some false starts" and a lack of

infallibility, "a weekly index of stock prices, with a monthly chart of previous years for comparison, furnishes as good a single barometer as we have available."

Chapters XIX to XXII are devoted to indexes of production, trade, the labor market and commodity prices. There is very little in them that is not purely descriptive in character. As with the chapters on the commercial services, the authors merely describe without any critical comment various governmental and private indexes that have been compiled either regularly or occasionally.

The appendices to this volume include various tables of statistical data, a bibliography, and "the forecasting record from November, 1918, to August, 1926, of four of the five forecasting agencies whose methods are described in Chapters V to IX," and also a forecast which Mr. Hardy himself made at the close of 1923. One is inclined to believe that these appendices will not help to "sell" business forecasting to business men!

This volume is not the first work of Mr. Hardy's which the present reviewer has been called upon to appraise. In reading what Mr. Hardy writes, one pretty generally has the feeling that his feet are on the ground, and this is so with the work now under review. There is an air of realism about Mr. Hardy's utterances in the field of business conditions which is not always found in books on this subject. This merit and the necessary concomitant of accuracy are among the virtues of *Forecasting Business Conditions*. The book has the further merit of being a concise summary of much of the important work that has been done in the field of business statistics. As such it must have value for many persons, from students to business men, who want an introduction to the subject. It has the grave fault of serious unevenness of treatment of the various subjects covered in its twenty-two chapters. As a result, even though the book does include a brief statement of the authors' preference among business cycle theories, followed by a chapter on "Practical Forecasting," which emphasizes the necessity of combining historical comparison and "cross-cut" analysis, one is left with a sense not only of lack of finality about the book but even of a lack of pointing up to anything in particular. Given the audience to which the book was addressed, this is unfortunate.

By the time Mr. Wallace has in two opening chapters indicated the need for business forecasting and stated the problem, and then followed this with the citation of the commercial forecasting services previously mentioned, he has written forty-five pages. There are only fifty more pages in the body of the book. They are for the most part devoted to examples of the manner in which business forecasts can be applied in manufacturing, selling, the forecasting of individual commodity prices, and in the field of investment. These chapters are a sort of "meditations" and contain much general advice of the type that is found in many American books, of a non-technical sort, on various phases of "management" problems. To the American who is at all familiar with either management literature or business cycle literature, the book will of course contribute little, as indeed the author would be the first to expect it to. His assumed audience is one of British business men. To these he is constantly saying that forecasting is not a substitute for judgment or for efficiency in other directions, but only an aid. It cannot be 100 per cent correct, but can perhaps better by

25 per cent the only half-accurate forecasts which hunches are likely to produce. The body of the book ends with a peroration on Britain, her wonderful history and the challenge which she must now meet from "young and virile nations," for world leadership in business. "She has no wish to shirk the challenge. . . . It is in the belief that business forecasting is just one aid to her endeavor which cannot safely be neglected, and in the hope that some of her business men may be persuaded to explore the possibilities of the subject, that this little book has been written. It is a small tribute laid at the feet of the 'Old Mother.'"

The missionary character of the book is thus obvious. Mr. Wallace, however, is not a fanatical missionary but altogether level-headed; almost too level-headed to make converts. At any rate, his level-headedness makes him realize, as did the first great missionary of the Christian church, that prospective and young converts must be given milk and not strong meat. There is accordingly not much strong meat in this volume.

F. E. RICHTER

American Loans to Germany, by Robert R. Kuczynski. Institute of Economics Series. New York: The Macmillan Company. 1927. xi, 378 pp.

Professor Kuczynski's study of *American Loans to Germany* is the latest of the investigations of post-war reconstruction sponsored and conducted by the Institute of Economics. It is divided into two quite different parts. The first and much shorter section of the book reviews the whole field of German foreign borrowing, from 1924 to the end of 1926. The inception of the Dawes Committee's work in 1924, and the prospect that a preliminary solution of the Reparations tangle would be reached, enabled Germany to re-enter the world's capital markets on an important scale for the first time since the War. The first step was the granting of English and American credits to the Gold Discount Bank, in April, 1924; then followed a number of short-term loans to industrial enterprises; then the Reparations (German External) Loan in October, 1924; and finally from July, 1925, to the end of 1926 a great flood of long-term loans to states and industries, cities and churches, and indeed to almost every type of organization.

In examining these borrowings, Professor Kuczynski begins by eliminating certain classes of loans entirely, either because data are not obtainable or for other reasons: namely, mark loans, loans to American companies and to the Saar basin, and loans for the consolidation of pre-1924 municipal debts. He confines himself to the foreign currency loans to Germany proper, both publicly and privately placed; and of these latter two divisions he deals principally with the first. The volume of loans which were privately placed is extremely difficult to gauge, since public records are usually not available. The evidence Professor Kuczynski has been able to collect indicates that their volume amounted, in the period 1924–1926, to the equivalent of about 130 million dollars, of which two-thirds was short term; but the actual aggregate was undoubtedly much larger. The volume of loans which were publicly placed, on the other hand, can be ascertained with more assurance. In the period 1924–1926, it amounted in dollar equivalents to 885 millions. But this very large figure is of course only the *nominal* capital of the loans. Since most of the loans were floated below par,

both the cost to the lender and especially the proceeds to the borrower were distinctly smaller. Nor does it represent the amount outstanding abroad at the end of 1926. Some redemptions have occurred, and many of the securities themselves have undoubtedly been bought back by Germans, thus altering the total outstanding to an extent that cannot even be guessed at. The distribution of the total of loans by lending countries was as follows: United States, 65.6 per cent; England, 14.2 per cent; Holland, 9.2 per cent; Switzerland, 5.4 per cent; seven other countries under 2 per cent apiece. The distribution by principal groups of borrowers was: federal, state and local governments, 46 per cent; industrial and commercial corporations, 30 per cent; public utilities, 15 per cent; credit, savings, religious and welfare organizations, 9 per cent.

The aspect of the borrowings which is of greatest general interest at present is the question of control over their volume, as exerted both by Germany herself and by the lenders. The German Government attempted to limit this volume at an early point, and by a law of March 21, 1925, required the municipalities and associations thereof to submit their loan proposals to a federal Advisory Council (the celebrated *Beratungsstelle für Auslandskredite*) for approval. The states have also voluntarily submitted their proposals. Of a total of 411 million dollars of loans submitted to the Council, 30 per cent have been disapproved, chiefly those of the municipalities. The remainder, which were approved, constitute less than a third of the total of loans floated in 1924–1926. The German Government thus checked the issue of foreign public loans to some extent, but did not control private loans at all; and its sphere of action has therefore been much smaller than is generally realized. Control by the governments of the foreign lending countries has also been very limited. The American Government, in 1924–1926, barred one loan (the potash loan), which afterward went to England, and the British Government prevented the public offering of German loans until the fall of 1925, but did not interfere with private sales. The Amsterdam Stock Exchange prevented official listing, but did not hinder the public offering of such loans.

The second section of the book, which forms nearly nine-tenths of the total space, consists of an elaborate description and classification of the forms and terms of those publicly offered loans to Germany which were made by the United States alone. No two issues were entirely alike, however, and it seems almost impossible to find any general characteristics in them, or to draw any general conclusions—unless we except the conclusion reached at one point, that in many cases the prospectuses of the loans exaggerated the assets of the borrowers, and did not tell the whole truth about their liabilities.

Finally, with respect to the moot question of productivity, Professor Kuczynski concludes that in the main the loans will contribute to a revival of German economic activity. But he points out that this still leaves unsolved the real problem, of whether the use of the loans will yield the foreign exchange necessary for their current service, and of whether any current surplus will result which can be applied to Reparations transfers. To this no unequivocal *a priori* answer can be given.

JAMES W. ANGELL

Columbia University

The American Negro, A Study in Racial Crossing, by Melville J. Herskovits. New York: Alfred A. Knopf. 1928. xiv, 91 pp.

The presence of ten million Negroes in the midst of a population of one hundred million whites offers an exceptional opportunity for the students of races and race mixture. Dr. Herskovits, a student of Professor Franz Boas, anthropologist of Columbia, has taken advantage of this opportunity and he has measured over fifteen hundred persons, who are commonly called, in our loose terminology, "Negro." Some of these were seen in the Harlem population of New York City, some in a visit at Howard University, and some in a rural community in West Virginia. Nearly a quarter were believed to be of unmixed Negro blood; another quarter were more Negro than white, with or without the admixture of Indian; another quarter were supposed to be about equally of Negro and white blood, with or without Indian mixture; and the remainder inclined strongly toward the white side. Negroes have been measured before, of course, but so large a body of mixed bloods, of which the racial stock is more or less known, has not been hitherto analyzed. Herskovits points out the comparatively small variability of the mixed Negroes in respect to certain of the traits, such as sitting height, height of ear, total height of face and cephalic index. The mixed American Negroes generally exceed in variability the West African tribes that have been measured but are sometimes less variable than "Old Americans." It may be pointed out, in criticism of the argument, that, first, Old Americans are a hybrid lot and, second, that mixed populations are not variable in all characters but only in those in which there is a genetic difference. Thus in nose width, lip thickness, hair curl, skin color, the mixed American Negroes would be found to be more variable than Old Americans. The conclusion that Herskovits draws from this low variability is that a new physical type is arising and the cause of its origin is a mate selection which tends to keep the mixed population near its average point. In the final chapter Herskovits discusses the significance of the Negro for the study of race. He regards him as constituting a stable type but he pleads: "Let us not think of him as a new race, nor as anything but the homogeneous group he constitutes." Yet it is not clear what a race is but a homogeneous group that is more or less distinctly cut off from other groups. The argument against calling the new type a race would seem to be that the recognition of racial distinctions often leads to unhappy relations between the races. We shall not quite reach the bottom of the matter until we understand the basis of race antagonism or, to put it the other way round, of a strong instinct for homogeneity in a population.

C. B. Davenport

Carnegie Institution, Cold Spring Harbor

Seventieth Report of the Commissioners of His Majesty's Inland Revenue for the Year Ended March 31, 1927. London: His Majesty's Stationery Office. 110 pp.

One's first reaction to this report is appreciation of its brevity and for the large amount of data presented in a rather brief space. There are but 110 pages,

chiefly tables with a minimum of interpretation. For the reader familiar with British finance this is adequate. There is no attempt to show that the receipts of the year just ended are due to the virtues of the Government in power; that the country is prospering under the present administration; that any one group is paying more or less than its share of the burdens; that the Department recommends certain changes. The facts are listed—no more, no less.

On the other hand, all readers but the expert need more. No one can imagine the general public learning much of their government's affairs from such a presentation. Even more than is the case in the United States an explanation of the facts is needed. Very few can or will read with profit such a document.

An outstanding feature is the size of the Exchequer receipts, the gross amount for the year covered being £402,047,000 and the net amount (gross less repayments) being £398,048,110. This is a slowly diminishing amount, the decline being steady since the high level of £717,550,771 in 1919–20. Yet the percentage cost of collection is rising. This is figured very properly on gross amount collected by Inland Revenue officials for all departments. For 1926–28 this was 1.61 per cent on a gross of £448,209,415, yet in 1916–17 the rate was .69 per cent on a gross of £356,172,765, and in 1920–21 was only 1.13 per cent on a gross of £684,536,207. The Internal Revenue Bureau of the United States had a cost of 1.15 per cent in 1926–27 incurred in the collection of $2,865,683,129. With us also this cost has risen from a low of .33 per cent in 1918 to a maximum of 1.44 per cent in 1925 followed by a decline to the percentage just given for 1927.

Of the total receipts by far the largest amounts come from the income tax (£230,135,868) and the accompanying super-tax (£66,295,611). Next are the Estate duty together with the Legacy and Succession duties, etc., with a total of £67,431,791. Stamp duties come next (£24,869,536) followed by a number of others of less importance.

With some modifications in rates the income tax yields a little less than in the past, but the reduction is not great—only from £314,374,524 in 1922–23. In the same period the yield from the super-tax has been increasing. Effective reductions have been made in the standard rate of tax, but not in the super-tax, especially in the higher brackets. This is an interesting contrast to the policy of our own Government. The British standard rate was 6s in the £ from 1920 to 1922, then 5s, next 4s, 6d and is now 4s in the £. Rates on super-tax, on the other hand, have been only slightly changed since 1920–21. There have been no reductions at all on the incomes above £20,000, and only moderate ones on incomes from £6,000. Even below £6,000 the changes are not great.

In another particular British policy has been different from ours. Rates of taxation on estates have not been reduced. Changes in these rates since the end of the War have been sharply upward with no changes since June, 1925. Since that time the rate has been 40 per cent where the net principal value of the estate exceeds £2,000,000. Under these rates the yield has been steadily increasing, that for 1927 being the highest to date. Similarly the yields from the legacy duty and the succession duty have been increasing. In America revenues have been so large as to permit a reduction in rates, but Great Britain has been less fortunate. In the reductions that have been effected she has been more fortu-

nate than we, since she has not conceded so much to the belief that high rates reduce yield.

Returns are now available in summarized form for 1927–28, but will probably show little change from 1926–27. But by 1929, when the returns for 1928–29 are accessible, interesting comparisons should be possible. Mr. Churchill's new budget will affect both the form of the accounts (making some comparisons difficult to present) and the taxes imposed.

ERNEST MINOR PATTERSON

University of Pennsylvania

Statistical Methods for Research Workers, by R. A. Fisher. Second Edition, revised and enlarged. Edinburgh: Oliver and Boyd. 1928. xi, 269 pp.

The first edition of this unique work was reviewed in this JOURNAL for September, 1927. In the new edition the table for evaluating the significance of intraclass correlations is amplified and an improved method of calculating the ordinates of a trend curve is introduced. However, the most important addition is a chapter on "The Principles of Statistical Estimation," that is, of the choice of the "statistic" to be calculated from the data for measuring the property under consideration. This chapter points out some pitfalls whose existence will surprise even hardened statisticians. For example (page 255), the method of moments appears to be a poor way of fitting the Pearsonian curves in most cases.

Some experience with the first edition has suggested that university teachers should note the following points in order to help students to appreciate Dr. Fisher's remarkable and useful work.

Proofs are omitted and there is no supply of problems for students to work. As a textbook, consequently, the volume requires extraordinary activity on the part of the teacher. The style includes such peculiarities as the use of S for summation, which is now common among British statisticians though everybody else in the world, even in England, uses Σ. Of the same type is the omission of the subscripts indicating variables of summation, which one would often like to see present for explicitness. The convention that tables and figures should be accompanied by rather full summaries apart from the text is not followed, and a diagram on page 260 looks upside down. It would be interesting to know what sort of creature masquerades as *Primula* or *Gammarus* while supplying data for examples. The reader having only a casual acquaintance with the theory and vocabulary of Mendelian inheritance will in places feel lost. But the examples no doubt are concrete enough to the biologists for whom the book was originally intended, just as Shakespeare's now unintelligible quips must have convulsed Elizabethan audiences. The absence of journalistic details so meticulously inserted by lesser writers should be held against such a work no more than George Washington's spelling impairs his reputation. An American using statistics will do well to work through the book, translate it into his own tongue, and change his habits accordingly.

HAROLD HOTELLING

Stanford University

Prohibition: Its Economic and Industrial Aspects, by Herman Feldman. New York: D. Appleton and Company. 1927. xv, 415 pp.

Professor Feldman in this study of national prohibition has limited himself to its economic results. The project was a hazardous undertaking, and he has shown courage in investigating a subject which so many social scientists have been unwilling to tackle. Among the difficulties facing a student of such social legislation are the absence of sufficient statistics and the task of interpreting the data which do exist. These difficulties obstruct the social scientists in many directions, but they especially beset the investigator of the prohibition question.

Professor Feldman has attempted to overcome the absence of data by means of the questionnaire and interview. Many of his conclusions depend, therefore, upon the trustworthiness of this method. He recognizes the perils of the use of the questionnaire and gives evidence of attempting to safeguard himself against them. His questionnaire, published in the book, shows careful planning and good judgment, and should be interesting to statisticians. By carefully explaining his plan of choosing the list of persons to whom the questionnaire was sent, he avoids the possible criticism of controlling the answers through the selection of addressees. Both in preparing the questionnaire and in treating the replies, he shows an objective point of view as well as scientific competence.

The conclusions reached by the author may be summarized as follows:

(1) The reduction of the consumption of liquor. The present supply of liquor arises from the diversion of legally distilled spirits, "moonshining" (manufacture in illegal establishments), smuggling of foreign liquors, and the making of wine, beer, and spirits in the home. Admitting that "there is much drinking going on today" (page 381), the author gives it as his opinion that when the quantities from all these sources "are added up, there is still reason to doubt if the amount available and consumed approaches anywhere near what would be the consumption of liquor today if it had proceeded on the pre-prohibition rate shown in the years 1900–1914, or even if it is near the 1914 figure" (page 53). He admits that "there are no authoritative figures . . . which would make our assertion more than a guess, and the reader may, therefore, incline to an opposite opinion without much danger of being disproved" (page 54). It is interesting to note that, in the period 1900–1914, the use of the three types of beverages—beer, wine and distilled spirits—was increasing more rapidly than population (page 33). This was the period before the wave of state prohibition, the higher taxes and wartime regulations. There is thus no foundation for the popular view that pre-prohibition temperance was increasing, if the annual per capita consumption may be taken as an index (page 383). This fact has a bearing upon the question of what the present probable consumption of liquor would have been in the absence of prohibition. Prohibition has "prohibited" to some extent even if the quantity now used is equal to the consumption before the amendment. The author has failed to prove statistically that national prohibition has reduced the total consumption of liquor, but his study of the available figures indicates that the "Wets" are unable to give a quantitative demonstration that it has not. This is progress.

Professor Feldman believes that the quantity of liquor diverted to illegal uses is

much over-estimated, and is inclined to accept the lower estimates of diversion made by the Prohibition Bureau because it is "closely in touch with the situation" (page 53). With reference to deaths from denatured alcohol, he shows from census figures that the number is greater now than in 1923, but less by about a half than in 1920 (page 401). He admits that there is evidence of "widespread making of wine at home," that "home distilling undoubtedly is carried on by many people," but thinks that the amount of home-brewing is over-estimated (page 127). As to arrests from drunkenness, deaths from alcoholism, and the proportion of charity cases due to intemperance, he shows that "none of these . . . is as severe as in pre-prohibition years" (page 391). But the "Wets" may derive some comfort from his finding that "the trend (of these statistics) since 1920 has been strikingly upwards" (page 391).

(2) The effect upon drink expenditure by the masses. He definitely states that the wage-earners and the great mass of the people "are paying out a great deal less on drink and all that went with it than they did before prohibition, or than they would today if we did not have it" (page 118). This conclusion is based not upon statistics but upon the observations of business executives in a position to know the condition of the workers, and upon the answers to a separate questionnaire prepared by the author and sent out by three insurance companies whose agents weekly or monthly call upon workers for their premiums (pages 125–126). He finds indirect evidence also of less spending on drink by this class, in the decrease, in certain centers, of the number of charity cases due to intemperance (page 133), in the number of instances in which wives come to get their husbands' pay checks to save them for home needs, and in the number of garnishments at pay day (page 132). Resting as it does upon indirect evidence and the observations of others, the conclusion is open to possible objection. However, he supports it further by giving reasons why present expenditure for liquor by workers should be less than formerly. These are: that the "boot-legger's" prices are high enough to reduce liquor purchases by workers (page 126), that the home-produced liquor is cheaper than the pre-prohibition article and requires less disbursement (page 128), that the passing of the saloon has done away with the excessive buying of liquor through the "treating" habit (page 130), and that the pay-checks are not cashed, as in pre-prohibition days in the saloons where much of the check-money was spent, but are now cashed by the savings banks, employers, and the stores (page 129).

(3) The effect of reduced liquor consumption upon other purchases and upon thrift. The author then proceeds to the inquiry: To what uses is the former liquor money being put? His answer is: "Part of the desire for the saloon's beverages has been diverted to milk, ginger ale, root beer, and other soft drinks; some benefit may have been reaped by the coffee trade; ice cream and candy stores have gotten a good deal of extra trade because of the closing of the saloons; cafeterias and small lunch rooms have taken over the food features." He finds that recreational features of the saloon are now partly met by radios, cars, and the "movies" (page 383). Some of the money has gone into savings deposits and industrial insurance, but he warns against attributing too much of present-day thrift to prohibition (page 384).

(4) The effect on production. Here again there is scarcely any statistical evidence. From the observations of business executives and others he reaches the following results:

(a) There has been a reduction since prohibition of the number of discharges from industry for drunkenness (pages 198, 397).

(b) The number of absences after pay day has markedly decreased although this change is attributed only in part to prohibition.

(c) No employers reported that industrial accidents had been increased by prohibition, and many industrialists as well as members of accident boards credited prohibition with a reduction of injuries (page 223). In the absence of satisfactory statistics, the writer states that prohibition can be only one factor in the decline of the accident rate (page 224).

(d) As to the effect upon the productive efficiency of the individual workers, the author is informed that on the whole, since prohibition, they are "of higher type, steadier, stronger, clearer-headed, more alert, and more efficient" (page 378). It is indicative of his caution that he opposes any attempt to estimate the effect of drinking or not drinking upon individual output on the basis of a few laboratory experiments. He particularly criticizes Professor Irving Fisher for the use of this method in the latter's book, *Prohibition At Its Worst*, in which that writer computes the money value of the estimated increase of the workers' output due to prohibition (page 240).

(5) The losses and disadvantages of prohibition. Against these alleged economic benefits of prohibition, the author places the losses which it has brought and strikes a balance. The losses or disadvantages are found in the scrapping of breweries and distilleries, the diversion of taxes from public treasuries into the profits of boot-leggers, the cost of enforcement, and the disappearance of markets for producers of hops and fine barley. With reference to the last point, however, he shows that, while these farmers have lost, dairymen and grape-growers have gained (pages 372–375). Striking his balance, he concludes that economically the disadvantages are not as great as the benefits. Nevertheless, he is far from complacent, asserting that the present benefits are insecure, and that the next few years will show whether they will continue.

Such studies of current problems as Professor Feldman's raise the question of their value. It is the opinion of the reviewer that this one is worth while. Though many of the results are inconclusive, the questionnaire method does something to fill the gap in our information caused by the absence of statistics. Moreover, by merely showing that certain propositions cannot be proved statistically, the book protects the public from the alleged statistics furnished by both sides in this controversy. Its chief value, however, doubtless, lies in the fact that it reveals the need for the systematic collection of useful industrial and economic data.

K. M. Williamson

Wesleyan University

Juvenile Employment and Labor Mobility in the Business Cycle, by Maurice B. Hexter. Boston: Massachusetts Child Labor Committee. 1927. xix, 111 pp.

Dr. Hexter has again demonstrated the feasibility and the significance of using modern statistical method for the measure and analysis of sociological data. Two years ago he published the results of his study of the fluctuations in the birth, death, marriage and divorce rates in the city of Boston, which he compared with the corresponding fluctuations in the business cycle. He has now utilized the records kept by the Boston school authorities as part of the administration of the Massachusetts Child Labor Law for a study of the trends and seasonal variations of juvenile employment, relating them to the ups and downs of business.

Since the law requires the issuing of different certificates to children of different ages and various degrees of educational qualification, when they first apply for permits to work and when they change jobs after the first hiring, Dr. Hexter has been able to set up six separate time series for the respective groups of children during the period from 1914 to 1928. These series have then been correlated with a number of well-known employment series and production indices.

A trend downward in the number of children employed in Boston during the thirteen years is clearly demonstrated. Since it has been accompanied by a growth in the population, it may be taken as an indication of factors militating against the employment of children. The author lists, as possible factors, protective legislation which restricts the employment of children, the reluctance of employers to take on young workers now that the high labor cost of that type of labor is becoming recognized, and the increased mechanization of industry with its decreased need for the services which children can perform. He does not essay, however, to assign a relative importance to these causes.

Dr. Hexter is not able to explain fully why one of the six series, an exception to the others, exhibits a slight increase in the numbers employed. This series relates to children who have come back for second permits but who still are obliged to attend night school. It is suggested that this series also will ultimately show a downward slant, and that the present upward slant is due to special conditions in 1919 which obscured the normal trend.

The responsiveness of young workers as well as adult labor to business conditions is seen in the expansion and contraction of all of these groups, but the cyclical fluctuation is much more decisive in the case of children who are changing jobs than in the case of those who are entering industry for the first time. The mentally retarded children—those who had failed to reach the sixth grade at the usual age—responded even less. These children were "sluggish" in their response to seasonal changes also. Dr. Hexter hazards a guess that this lack of response is due to their willingness to stay in industry at lower rates of pay than the abler workers will accept.

For readers who are interested in the interactions between school and work the most significant characteristics of the seasonal responses of juvenile workers are to be found in the June and September peaks for the issuing of new certificates. These peaks indicate the rush towards permanent or vacation work when the

school year is over and the preference for work to a return to school in September. In the two series composed predominately of retarded children, the peaks appear "in the winter and fall months with a spring rise." This is attributed to their desire "to make their exit from school at the earliest possible moment without regard to the terminal dates of the school system when they might be promoted to higher grades." This explanation of the tendency, which has been noted elsewhere by students of retardation, would be more readily acceptable if it did not require the assumption of a different distribution of birthdays for the mentally retarded!

The data have been assembled with great care, and enormous industry has been shown in the elaboration of statistical detail. The light which the study throws on the situation of children at work is excellent proof of the desirability of realizing Dr. Hexter's ambition for the establishment of a national juvenile employment index.

AMY HEWES

Mount Holyoke College

Profits in Bank Stocks, by H. W. Woodward. New York: The Macmillan Company. 192 pp.

This book is a salesman's argument in favor of the purchase of bank stocks. It is frankly partisan. Mr. Woodward does not believe there are any valid reasons why a prospective investor should not buy bank stocks. While he states that the purpose of the book is to educate the investor who is contemplating investment in bank stocks, anyone seeking a genuine analytical discussion as to why bank stocks should be purchased rather than public utility stocks, rails, or industrials will not find it.

The argument sums up about as follows: Bank stocks have low yields; so should be purchased for future appreciation in value rather than for present return on the amount invested in them. The investor may be assured that he will receive such appreciation in value because investors have enjoyed appreciation in the past. There is no analysis whatsoever of the factors that have determined past appreciation in value nor any reasons given as to why the past rate of appreciation should be projected into the future. Prospects of increasing competition among banks for deposits and the relation of this to future profits receive no attention. Nor on the favorable side is any mention made of the effects of branch banking on the future earning power of banks. When the book attempts to be analytical, it misinforms. For instance, the reader is told (page 21) that banks make money in both good and bad times; that they are not likely to be affected by the ups and downs of business because in depression they make up for the low interest rates they receive by a larger volume of loans, and in prosperity, they make up for a smaller volume of loans by higher interest rates.

The book contains some good homely advice to the inexperienced investor and contains some useful statistical information for several large banks and insurance companies.

C. R. CHAMBERS

Investment Research Corporation, Detroit

Negro Problems in Cities, A study made under the direction of T. J. Woofter, Jr. New York: Doubleday, Doran and Company. 192 pp.

One inevitably thinks of this study as a kind of continuation of the work which was incorporated in the older surveys of the practical aspects of the Negro problem by Atlanta University. It has much of the detachment, careful collection of data, and sound appraisal of results which characterized that excellent series. And since the Atlanta pamphlets are no longer appearing, it is well that the Institute of Social and Religious Research should have turned to a consideration of the facts of the urban conditions of Negroes.

Dr. Woofter was assisted in this study of living conditions of the Negro in sixteen cities in the North and South by three assistants, who divided the subject with him. Dr. Woofter first gives the general scope and purpose of the study, and discusses the problem of the Negro neighborhood, both as to its own peculiar situation and its relation to the white neighborhoods which it adjoins, or which, perhaps, in the course of Negro expansion, it eventually incorporates in its limits. Miss Headly then takes up one detail with a consideration of housing conditions in the cities studied, Mr. Daniel follows with a striking survey of the educational situation as far as public schools are concerned, while Mr. McGuinn discusses the recreational aspects of Negro habitation in cities.

In his introduction, Dr. Woofter discusses the need for a new analysis of the situation of the Negro in cities, not only in the North but in the South as well. From his discussion it appears that Negro migration is not alone from South to North. There have, in fact, been two kinds of Negro migration, one that of popular conception, but the other that from southern farm to southern city. The increase of the Negro urban population in the South during the decade of 1910–1920, which was 21.4 per cent as against a 3.4 per cent decrease in the Southern rural population, makes the point in striking fashion. Yet we have not been entirely wrong. For while the Negro city population has increased by a million and a half between 1900 and 1920, one million of this increase is shown to have been concentrated in the "metropolitan centers of the East and the Middle West." And although the natural rate of increase is low, the rise in Negro city population seems to continue, and the author feels that "it is apparent that the city movement continues."

As a result of this fact of growth friction has come. In all of the cities studied, Buffalo, Chicago, Dayton, Gary, Indianapolis, New York, Philadelphia (in the North), and Charleston, Knoxville, Lexington, Louisville, Memphis, New Orleans, Richmond and Winston-Salem (as southern cities), this friction has been more or less apparent. In Chicago, there were race riots, in Louisville and New Orleans and Indianapolis segregation ordinances, and everywhere some kind of social pressure to prevent the Negroes expanding the districts where they live. Yet it is of interest that Dr. Woofter found that far from most of the census enumeration districts had as much as 90 per cent of their population Negroes, while in a vast number there were not 90 per cent whites. There is, actually, less of hard and fast segregation than we think—it is only the dramatic instances of it that attract our attention and arouse our resentment. The results of segregation are next considered; the depreciation in property values (more fancied

than actual, I fear, in view of the profits to be made from renting places of living to Negroes, as shown in later chapters), the racial solidarity, the neglect and exploitation. The outstanding contribution of the book, perhaps, is the series of maps which accompany this chapter, and which graphically give the actual extent to which the Negro districts in these cities have grown from 1910 to 1925. These maps tell the story at a glance, and show, among other points, how the Negro districts are often not nearly so localized as we sometimes think.

Dr. Woofter closes by considering the form exploitation takes in these Negro neighborhoods; how more and more families are crowded onto a given acreage, how the lodger-evil arises, and the relation of density of population to death and the incidence of disease; and, finally, the types of neighborhoods in which Negroes live. It is to be regretted that in his discussion of the newer suburban developments the community at Jamaica, on Long Island, was not selected for special study.

In Miss Healy's section we are given the results obtained by house-to-house canvass; detailed consideration of the extent to which Negroes own their own homes, the numbers of rooms at the disposal of those living in the houses in the Negro districts, the use of the rooms and the equipment of the houses, and such details as the walks, toilet facilities, and the extent to which repairs are made. There is an excellent chapter that shows how the tremendous pressure due to segregation boosts the rents to heights which the white renter does not have to reach, and it is a comment on the impartiality of the study that the writer states "with the increase in the purchase of apartments by Negroes as an investment it seems probable that neither lower rents nor improved conditions in Negro neighborhoods will result from Negro ownership of rental properties"—results that those conversant with conditions in Harlem or Chicago's South Side will heartily endorse as another demonstration of the extent to which the economic motive does not know the color line!

Mr. Daniel's study of the school situation shows how the policy of segregation works out to the detriment of the Negro in education, especially in the South. Everywhere we find that where there is segregation, the policy of "just as good facilities for the Negro" seems to be a myth. And it is to be regretted that the study must show so many of the northern cities pursuing such a policy. But in the South the figures are appalling. The following table showing the per capita expenditure for white and colored children in the schools of four southern cities speaks for itself:

Item of Expense	*White*	*Negro*
Capital investment	$172.64	$61.59
Current expenses	48.96	29.16

His discussion of the standards and rewards of the two systems, Negro and white, do nothing to relieve the picture; thus, Lexington provided a minimum of $1,000 for white elementary teachers, a *maximum* of $900 for Negro elementary teachers. Or, there are 162 kindergartens for white children in eight southern cities, 8 for Negro children. And where there are schools that go beyond the eighth or ninth grade, or new buildings built, the crowding necessitates double and triple sessions.

The section on recreation by Mr. McGuinn is perhaps the least essential. The

same story is told, of course; segregation leads to discrimination. But one feels lacking the same scholarly detachment—finds too much of the crusader's zeal to allow this section to be in harmony with the scientific scrutiny of objective facts characteristic of the rest of the book. Mr. McGuinn sets standards of conduct in a fashion that is not convincing; I am afraid that the careful tabulations of the rest of the book do not accord with such statements as one that asserts that "the dancing of 60 per cent of the couples was obviously based on erotic thrill." Of course, Mr. McGuinn was dealing with discrimination in its most crass forms, and the picture he paints of the kind of gallery seats Negroes must take if they wish to see movies or go to the theatre, and the tricks that are played on them in those states where discrimination is legally outlawed, is not pleasant. But this portion of the book, scientifically, falls far below that of the rest of it.

The work, as a whole, however, gives an excellent impression, and should be of great value to students of the Negro problem. It is to be hoped that the Institute of Religious Research will inquire into other phases of it as well.

MELVILLE J. HERSKOVITS

Northwestern University

Mathematical Principles of Finance, by Frederick Charles Kent. New York: Second edition, McGraw-Hill Book Company, Inc. 1927. xiii, 177 pp., and compound interest and annuity tables, 214 pp.

Those interested in the mathematical principles of the theory of interest and annuities will find in Mr. Kent's book a welcome text. The fundamental formulae which the student must use in his work are developed from actual examples and are frequently followed by practical problems relating to the preceding theoretical discussions. The author calls particular attention to the methods employed in the derivation of interest and annuity formulae, to the reduction of annuity formulae for nominal rates of interest so as to obtain directly from tables the solutions for many problems which would otherwise require a logarithmic treatment, and to the ten place tables of compound interest functions for thirty-six rates of interest, nine of which are less than one per cent. Besides these tables a six-place table of common logarithms has been included.

Each chapter is followed by a number of miscellaneous exercises. Numerical answers are not given as is the custom in most textbooks, nor is Mr. Kent inclined to make liberal use of graphic presentation which would be most instructive for the student especially when dealing with logarithmic functions. For the convenience of the student the *Ten Place Interest and Annuity Tables* may be obtained with the revised edition of the text or in a separate volume if so desired.

R. VON HUHN

Municipal Savings Banks. Report of the Committee appointed to consider whether it is desirable to permit a further extension of Municipal Savings Banks. Presented to Parliament by the Financial Secretary to the Treasury by command of His Majesty, January, 1928. London: His Majesty's Stationery Office. 75 pp.

In pre-war Britain there were no Municipal Savings Banks to encourage the honest workman in the exercise of thrift. Saving there was, nevertheless, in

sufficient measure to justify two great national systems of savings institutions, one of private origin, comprising the affiliated Trustee Savings Banks, the other, a special creation of the Post Office authorities. The only vital distinction between these two systems in recent years has been in respect to historical origin as both collected savings over large areas of Britain and remitted these funds to the British Treasury, where they were invested in the national debt. In addition to these two national systems, there were numerous minor institutions which also dabbled in the peoples' savings, but none seriously threatened their monopoly until the Municipality of Birmingham organized and operated a Municipal Savings Bank under authority of one of those numerous war-time legislative acts which let down the bars in so many countries and in such unfortunate directions. Of late the desire of other municipalities to imitate Birmingham has become sufficiently pressing to call for a Parliamentary investigation of the entire subject of Municipal Savings Banks in that country.

The report of the committee, which has recently become available, is adverse to the further extension of Municipal Savings Banks beyond the limits of enterprising Birmingham. In reaching this conclusion, the committee was duly influenced by the consideration that other municipalities might not adhere so rigorously to sound banking practice as Birmingham. At the same time it was not at all unmindful of the fact that a repetition of the brilliant success of the Birmingham experiment in other municipalities might prove as unpalatable an event to His Majesty's Treasury as would a series of failures to depositors. The Trustee Savings Banks and the savings department of the Post Office form a complete system by which the annual increments of new savings deposits of the country are drained off to the British Treasury where they help to float a rather unwieldy national debt. At least, such is the situation outside of Birmingham. In that city, the committee is convinced, the very success of the Municipal Bank has been at the expense of the older institutions with the result that funds which would otherwise have become available for application to the national debt have been diverted to municipal uses.

Entirely aside from the interesting conflict between local and national interests which gave rise to the appointment of the committee and dominated their conclusions, the report furnishes a concise and valuable account of the institutions which specialize in savings deposits in Great Britain. From an American point of view, the most interesting feature is the simplicity of the system and completeness with which such funds are diverted to governmental use. This applies fully as much to the Birmingham bank as to the older institutions, for by far the greater portion of the funds of that bank are loaned to the Municipality. Statistics on savings deposits in Great Britain are published in the Appendix to the report.

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